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(6) MODELING AND PERFORMANCE OPTIMIZATION OF

LARGE-SCALE DATA-COMMUNICATION NETWORKS.

by

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Abstract

Modeling and Performance Optimization of Large-Scale Data-Communication Networks

By Francis D. Gorecki

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A new theory of modeling, analysis, and performance optimization is developed and illustrated for large-scale, data-communication networks. In a novel and significant departure from existing problem formulations, channel and network models are presented which include the effects of channel distortion, nodal processing errors, and protocol dependent errors. All of these effects are of critical importance in practice, but cannot be readily incorporated into other models. Information theoretic techniques are developed, and used to characterize network capacity and message integrity as a function of protocols, routing policies, channel capacity assignment, and channel distortion. Flow assignment techniques which optimize network performance with respect to certain classes of physically important performance metrics are formulated and employed to derive new centralized and distributed routing algorithms.

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INTRODUCTION

1.1 Background

In the last decade, large-scale, data-communication networks have both expanded across continents into globally-distributed systems interconnecting major computer facilities, and contracted into office complexes, interconnecting laboratories and offices and their associated instruments and minicomputers. Examples of the globally-distributed networks are: ARPANET, a U.S. Department of Defense computer network [1]; TYMNET, a specialized U.S. based common carrier network with links to Europe [2]; and TRANSPAC, the French electronic mail system [3]. An example of a local-area network is ETHERNET, a system for carrying digital data packets among locally distributed minicomputers [4].

The basic purpose of large-scale computer-communication networks is to share computing resources among geographically distributed users. The resources may be databases, computer power, specialized hardware, or specialized software. (The phrases data-communication and computer-communication are considered synonymous for the purposes of this research and are used interchangeably throughout the text.) The large-scale network may be viewed as a collection of nodes and links which may be divided into a communications subnetwork and a user-resource subnetwork as shown in Figure 1.1 [5]. The communication subnetwork provides message

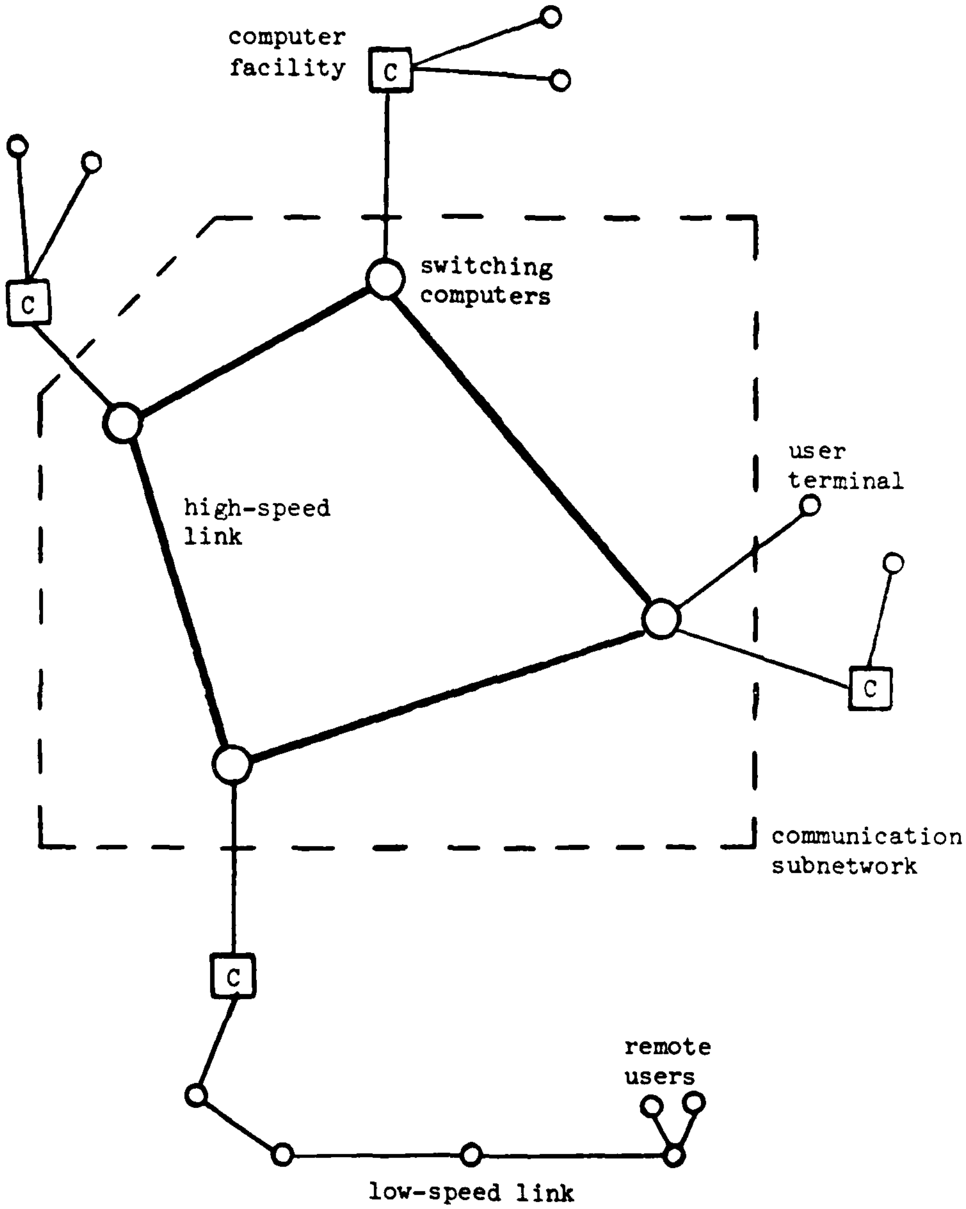


Figure 1.1 A network representation showing the communication subnetwork, the user-resource subnetwork, switching computers, and high-speed links

formatting, routing, flow control, and data integrity procedures for the user-resource subnetwork. Typically, the switching computers of the communications subnetwork are tied to a backbone of high-speed, point-to-point terrestrial links. All entry and exit for the network passes through the switching computers. The users at their terminals access distributed computing resources via the communications subnetwork. The switching computers of the communications subnetwork establish the necessary interconnections to link user and resource, or source and destination. The communication networks can be categorized by the type of linkage employed: circuit-switched; message-switched; and packet-switched. The circuit-switched networks establish dedicated paths for a given user-resource pair. The path is allocated for the duration of a user-resource session. (A different path may be used on a succeeding user-resource session, but it will be dedicated for the duration of the session.) This is the familiar mechanism used in telephone systems. Message-switched and packet-switched sytems, on the other hand, do not establish dedicated channels for simultaneous use by the user-resource pair. In message-switched systems, the message moves from a user (or source) node to the next node on a path selected according to a routing algorithm. The message is moved through the network in this store-and-forward fashion until it reaches the resource (or destination) node. A packet-switched system is similar in its use of the store-and-forward technique. A message from a user (or source node) is divided into smaller units or packets. The packets move through the communications subnetwork in a store-and-forward manner until they reach their destination. At the destination the message is reconstructed from the packets. Because each user or resource is not directly connected to all others, messages and packets pass through several links and switching computers before reaching their destination. (In general, the terms packet and message may be used interchangeably. If there is a restriction to packet or message format in the research, then that will be noted in the pertinent text material.) The rules for making the link connections, choosing the paths, and controlling the network traffic involve the use of computer-communication protocols and routing strategies [6,7]. The strategies employed may be chosen to optimize particular network performance measures. Network performance measures include message volume, link or channel utilization, average message delay, message or data integrity, and processor or buffer utilization. For example, the message arrival characteristics in a computer environment are bursty, hence the packet-switched approach provides a low-cost network solution by minimizing the number of channels required to connect a large number of users to geographically distributed computer facilities and to each other. Another tangible advantage of packet-switched networks is their ability to accommodate a broad spectrum of data rates. In a circuit-switched network, the communicating devices fall into mutually exclusive data rate categories. Packet-switching accommodates these diverse data rates through the communication subnetwork/user-resource subnetwork interface.

Theoretical studies have been directed to the network flow problems and the problems of delay analysis, routing assignment, topological design, and protocol impact [8,9]. Open problems still exist in determining optimum message routing strategies and the effects of channel distortion and nodal processing or protocol-dependent errors on network performance. The problem of optimum message routing is complicated because the network is a system of interacting queues and because of the large number of network variables involved. Similarly, the effects of channel distortion and nodal processing or protocol-dependent errors have resisted analysis because of an absence of a theoretical framework in which to operate and because of the overwhelming number of variables involved in analyzing a large-scale data-communication network.

1.2 The Problem of Data Integrity and Flow Assignment

Due to the complexity of the theoretical formulation, previous investigations have ignored the problem of message corruption both in the network performance analysis and the establishment of suitable routing doctrines [5,8]. The canonical network models have assumed distortionless communication of messages. Derivative performance analyses and routing algorithms are, therefore, best case results. In practice, of course, message distortion is encountered [10]. The existence of 'real world' sources of error have led system designers to use various techniques, such as computer-communications protocols with error correct/detect features, to maintain message integrity. This sort of practical solution to the message corruption problem exacts a system performance penalty, viz., message delay is increased due to the implementation of the requisite protocols. While protocol models do exist, they are too complex to be useful in modeling large-scale networks [11], and are limited to single channel behavior.

Thus, what is required for successful network modeling is a general methodology for analyzing the broad class of network problems relating to data integrity and its relationship to system performance in large-scale networks. The results of this general approach should then drive the routing assignment doctrine. (By the way of a parenthetical comment, it should be noted that:

1. Successful operation of a data communication network is critically dependent on the provision of an adequate routing algorithm.
2. The choice of routing algorithm strongly influences network performance.
3. Many Ph.D. dissertations have been devoted exclusively to development of a specific routing policy. (Few of these techniques have been implemented.)

This research has developed the needed general approach to message corruption in computer-communication networks and linked that approach to relevant routing strategies. The general approach combines both information-theoretic and queueing-theoretic techniques. The resulting routing strategies include both centralized and distributed policies.

1.3 Outline of Dissertation

In the remainder of Chapter 1, the basic problem formulation for network modeling is presented along with the stochastic 'network of queues' model. Chapter 2 develops the basic information-theoretic concepts and their extensions for data-communication network modeling, viz., the theory of rate distortion functions suitably enhanced and strengthened for multiple user network theory. Chapter 3 develops relevant routing strategies which optimize network performance with respect to certain classes of physically important performance metrics. Both centralized and decentralized algorithms are presented. Additionally, in Chapter 3, network examples in which the most significant of the routing algorithms have been used for flow assignment are presented. In Chapter 4, computational results derived from the application of the theory developed in Chapter 2 are presented as well as a network example which illustrates the results of the preceding chapters. Finally, conclusions and directions for future research are presented in Chapter 5.

1.4 Network Modeling

The purpose of this section is to provide a brief review of the principle modeling methodologies for large-scale, data-communication networks. Stochastic analysis, especially queueing theory, has been used to derive the bulk of the theoretical results in the analysis and synthesis of computer-communication networks. The first models are due to Kleinrock and were formulated in the early 60's [12]. The queueing-theoretic approach has had a great deal of success in modeling networks although it does not deal with a number of difficult problems such as channel noise, finite nodal storage, dynamic routing and stability, and flow control procedures. The basis for the model is the single server queueing system with exponentially distributed job service and interarrival times. This simple case is abbreviated as an M/M/1 system where the M/M stands for Markovian distribution of job interarrival and service statistics and /1 denotes a single server. (Other job statistics have been studied e.g. G/G/1, general distributions of both job service and job interarrival times. The general theory is quite complex and has not been successfully applied to networks [13].)

1.4.1 M/M/1 Queueing Models

Consider the server queue system shown in Figure 1.2. Customers arrive to the system with an average rate of λ customers sec. The customers wait in the queue until they are serviced. The average service time is $1/\mu$ sec. While waiting in the queue the customers are serviced on a first-come-first-served (FCFS) basis. In the steady state, the customers depart the system with an average rate λ. To make the discussion more precise, let $A(t)$ and $B(x)$ be the probability distribution functions of the interarrival and service processes, respectively. Then for M/M systems,

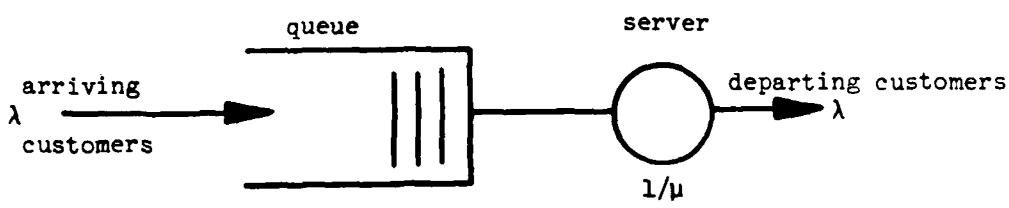


Figure 1.2 Model for M/M/1 queueing system

$$A(t) = 1 - e^{-\lambda t}, \ t \geqslant 0 \tag{1.1}$$

and

$$B(x) = 1 - e^{-\mu x}, \ x \geqslant 0 \tag{1.2}$$

Denoting E[•] as the expectation operator,

$$E[t] = 1/\lambda \tag{1.3}$$

and

$$E[x] = 1/\mu \tag{1.4}$$

As noted previously, λ represents the average arrival rate and $1/\mu$ the average service time for the customers.

Equation (1.1) is, of course, the interarrival distribution for a Poisson counting process [14]. The probability of k arrivals in an interval (T,T+t) for a Poisson process is

$$\Pr[k \text{ arrivals in } t] = (\lambda t)^k \frac{e}{k!}^{-\lambda t} \tag{1.5}$$

The differential-difference equations corresponding to the M/M/1 system are easily shown to be [13, p. 74],

$$\frac{dP_k(t)}{dt} = -(\lambda + \mu)P_k(t) + \lambda P_{k-1}(t) + \mu P_{k+1}(t) \tag{1.6}$$

for $k \geqslant 1$, and

$$\frac{dP_o(t)}{dt} = -\lambda P_o(t) + \mu P_1(t) \tag{1.7}$$

for $k = 0$, where

$$\Pr[\text{No. of customers in system at time } t \text{ is equal to } k] \equiv P_k(t) \tag{1.8}$$

This set of coupled equations may be solved by first taking the z-transform and then the Laplace transform of Eq. (1.6) and Eq. (1.7). The general solution is obtained by inverting the combined Laplace and z-transform transfer function. The solution is an infinite series of modified Bessel functions of the first kind of integer order. Understandably, the complete general solution is too unwieldy for network modeling applications.

Fortunately, the probabilities $P_k(t)$ have steady state limiting values. Defining

$$p_k = \lim_{t\to\infty} P_k(t) \tag{1.9}$$

and setting the left-hand terms of Eq. (1.6) and Eq. (1.7) to zero leads to the steady state solution for the system,

$$p_k = p_o\left(\frac{\lambda}{\mu}\right)^k \tag{1.10}$$

where $k \geq 0$. The necessary and sufficient condition for ergodicity is simply

$$\lambda < \mu \tag{1.11}$$

Applying the conservation of probability

$$\sum_{k=0}^{\infty} p_k = 1 \tag{1.12}$$

leads to

$$p_o = \left[1 + \sum_{k=1}^{\infty}\left(\frac{\lambda}{\mu}\right)^k\right]^{-1} \tag{1.13}$$

Defining the system utilization as

$$\rho = \lambda/\mu \tag{1.14}$$

allows the equilibrium solution to be expressed as

$$p_k = (1 - \rho)\, \rho^k \tag{1.15}$$

Certain important system parameters may be quantified after the introduction of Little's result. Little's result may be stated as: at equilibrium, the average number of customers in a queueing system is equal to the average arrival rate of customers to that system, times the average time spent in the system [15]. Denoting the average time in the system as T and the average number of customers in the system as $\bar{N}$, Little's result may be written as

$$\bar{N} = \lambda T \tag{1.16}$$

From Eq. (1.15), the average number in the system is easily shown to be

$$\bar{N} = \sum_{k=0}^{\infty} k p_k = \frac{\rho}{1 - \rho} \tag{1.17}$$

The average time in the system can then be found using Eq. (1.16) and Eq. (1.17) as

$$T = \frac{\bar{N}}{\lambda} = \frac{1}{\mu - \lambda} \tag{1.18}$$

The average delay as expressed in Eq. (1.18) is a key result which will be used repeatedly in this research.

1.4.2 Network of Queues Model

The M/M/1 system described in the previous paragraph can be considered as a single node system. One can also imagine multiple node systems in which a customer queues and receives service successively at several nodes. If the messages or packets of the previously described store-and-forward networks are considered as customers, and the switching computers and communication channels are viewed as FCFS queues with associated servers, then the queueing model for a store-and-forward network is a multistage tandem queue [8].

The simplest example of a tandem queue is the system of two servers shown in Figure 1.3, which was originally studied by R. R. P. Jackson in the 1950's [16]. The system consists of the first node, an M/M/1 queue with service-rate μ_1, feeding a second queueing system with service rate μ_2. Defining the stationary joint probability state vector of the system as $p(k_1,k_2)$ where

$$\Pr[k_1 \text{ customers in first system and } k_2 \text{ customers in the second system}] \equiv p(k_1,k_2) \tag{1.19}$$

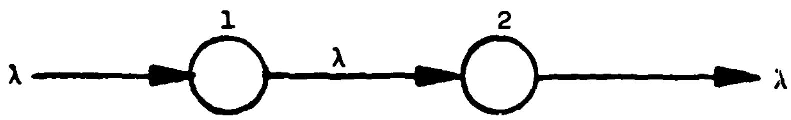
Jackson showed the joint probability was simply the product of the state probabilities for two independent M/M/1 queues, i.e.,

$$p(k_1,k_2) = p_1(k_1)p_2(k_2) = (1 - \rho_1)\rho_1^{k_1}(1 - \rho_2)\rho_2^{k_2} \tag{1.20}$$

In 1956, P. J. Burke and E. Reich proved that the output of an M/M/1 system was also Poisson [17]. These two results then lead to the conclusion that the joint probability state vector for any feedforward queueing system is equal to the product of the marginal distributions, namely,

$$p(k_1,k_2,\ldots k_N) = \prod_{i=1}^{N} p_i(k_i) \tag{1.21}$$

(a)



(b)

Figure 1.3 The two-node tandem net; (a) Details of the two-node tandem net; (b) Network representation

Equation (1.21) is also valid for systems with m servers at each node, i.e., M/M/m systems [18]. The feedforward systems are special cases of feedback networks. (In a feedback hetwork customers are allowed to return to previously visited nodes.) J. R. Jackson studied feedback networks in which external arrivals and departures were permitted [18]. The model considered consists of N nodes, the ith node contains m_i servers each with service rate $1/\mu_i$. Poisson arrivals are allowed to enter the ith node from outside the network with rate γ_i. After receiving service at the ith node a customer goes to node j with probability r_{ij}. The total arrival rate to the ith node can then be written as

$$\lambda_i = \gamma_i + \sum_{j=1}^{N} \lambda_j r_{ji} \tag{1.22}$$

Jackson's result for the joint equilibrium probability distribution for the network is

$$p(k_1,\ldots k_N) = \prod_{i=1}^{N} p_i(k_i) \tag{1.23}$$

These results can be extended to closed Markovian queueing networks in which a fixed population of customers circulate within the network. Again, the results are product form solutions [19,20].

The foregoing results would be directly applicable to the store-and-forward data communications network except that in such network there is dependence among the interarrival times and the service times. The dependence can be seen by considering the two-node system of Figure 1.3. The interarrival time between two successive messages (or packets) entering node 2 can never be less than the service time for the first of those messages [5, p. 321]. Therefore the arrival distribution to the second node is not Poisson and the results of Jackson, Newell, et al. do not apply. To circumvent this difficulty, Kleinrock invoked the Independence Assumption, to wit: Each time a message (packet) is received at a node within a network, a new length is chosen independently from the probability density $p(x) = \mu e^{-\mu x}$. In effect, each queue in a packet network is assumed to be an independent M/M/1 system with an arrival rate of λ_i. This assumption has given good agreement between theory and practice in networks of moderate connectivity and forms the basis of the research developed here.

1.4.3 <u>Delay Analysis</u>

Perhaps the single most important network performance measure is message delay, either average for all messages or end-to-end for a specified user-resource (source-destination) pair. The purpose of this section is to calculate appropriate expressions for both the average message delay T, and the end-to-end delay T_{ab} where a represents a source node and b represents a destination node.

Consider a N-node network with M directed links. The paths taken by messages originating at node j and destined for node k is denoted by π_{jk}. The ith link with capacity C_i is in the path π_{jk} if messages from j to k ((j,k) traffic) utilize that link. The traffic requirements for the network are denoted by an average of $\{\gamma_{jk}\}$ messages/sec. where j is a source node and k is a destination node

The average flow in the ith channel λ_i is related to the message flow rates of all paths which include the ith channel

$$\lambda_i = \sum_{j} \sum_{k} \gamma_{jk} \quad jk: C_i \varepsilon \pi_{jk} \tag{1.24}$$

Defining Z_{jk} as the average message delay for a message originating at j destined for k, then the average for all messages can be expressed as

$$T = \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{\gamma_{jk}}{\gamma} Z_{jk} \tag{1.25}$$

where

$$\gamma = \sum_{j=1}^{N} \sum_{k=1}^{N} \gamma_{jk} \tag{1.26}$$

Denoting T_i as the average delay encountered in waiting for and using the ith channel it is possible to express Z_{jk} as

$$Z_{jk} = \sum_{i:C_i \varepsilon \pi_{jk}} T_i \tag{1.27}$$

Combining Equations (1.25) and (1.27) yields

$$T = \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{\gamma_{jk}}{\gamma} \sum_{i:C_i \varepsilon \pi_{jk}} T_i \tag{1.28}$$

Interchanging summations and observing the conditions on the (j,k) pairs and using Eq. (1.24) gives the desired result for T[5, p. 321],

$$T = \sum_{i=1}^{M} \frac{\lambda_i}{\gamma} T_i \qquad (1.29)$$

Using the results of section 1.4.2, the ith channel is now represented as an M/M/1 system with Poisson arrivals at rate λ_i and exponential service times of mean $1/\mu C_i$.

The solution for T_i is then given by Eq. (1.18)

$$T_i = \frac{1}{\mu C_i - \lambda_i} \qquad (1.30)$$

Substitution of Eq. (1.30) into Eq. (1.29) produces a simple form for the average message delay

$$T = \sum_{i=1}^{M} \frac{\lambda_i}{\gamma} \left(\frac{1}{\mu C_i - \lambda_i}\right) \qquad (1.31)$$

The appropriate expression for the average end-to-end delay may be stated after defining the path indexed message routing variables $\{r_\ell(j,k)\}$ and the path and link indexed routing variables $\{r_\ell^i(j,k)\}$, following the approach of [21]. The path indexed routing variable $r_\ell(j,k)$ is defined as being the fraction of (j,k) traffic which is routed on the ℓth shortest (j,k) path. The set of links comprising the ℓth shortest (j,k) path is denoted by π_{jk}^{ℓ}. The path indexed message routing variable $r_\ell(j,k)$ will have the same value in all the links in the set π_{jk}^{ℓ}, and, in general, is the quantity of interest. Occasionally, however, it is necessary to denote the message routing variable for the ℓth shortest (j,k) path and for a specific link in the set π_{jk}^{ℓ}. To accommodate this requirement, the path and link indexed routing variable $r_\ell^i(j,k)$ is introduced.

The notation $r_\ell^i(j,k)$ denotes the fraction of (j,k) traffic on the ℓth shortest (j,k) path traversing link i. For example, assume the fourth shortest (j,k) path was made up of links 9, 10, and 16. Then the path indexed message routing variable is denoted as $r_4(j,k)$. The path and link indexed message routing variables are denoted by $r_4^9(j,k)$, $r_4^{10}(j,k)$, and $r_4^{16}(j,k)$. Of course, the numerical values of $r_4(j,k)$, $r_4^9(j,k)$, $r_4^{10}(j,k)$, and $r_4^{16}(j,k)$ are equal. The set of all links involved in carrying traffic from node j to node k [(j,k) traffic] is denoted by π_{jk}.

Non-negativity of flow and conservation of flow require that

$$r_\ell(j,k) \geq 0 \tag{1.32}$$

and

$$\sum_\ell r_\ell(j,k) = 1 \tag{1.33}$$

for all ℓ, j,k in Eq. (1.32) and for each (j,k) pair (each (j,k) pair is a commodity) in Eq. (1.33). Similarly, for the path and link indexed message routing variables $\{r_\ell^i(j,k)\}$, non-negativity of flow requires that

$$r_\ell^i(j,k) \geq 0 \tag{1.34}$$

for all i, ℓ, j, k in Eq. (1.34).

Note that

$$r_\ell^i(j,k) = r_\ell(j,k) \tag{1.35}$$

for all $i \epsilon \pi_{jk}^\ell$ and for each (j,k) pair in Eq. (1.35).

Using the above definitions, the end-to-end average message delay for the (a,b) source-destination node pair may be expressed in two ways. First, using the path indexed message routing variables the end-to-end delay is expressed as

$$\sum_{\substack{i\epsilon\pi_{ab} \\ \ell\epsilon\pi^{\ell}_{ab}}} \frac{r_{\ell}(a,b)}{\mu C_i - \lambda_i} = T_{ab} \tag{1.36}$$

Second, using the path and link indexed message routing variables the end-to-end delay is expressed as

$$\sum_{\substack{i\epsilon\pi_{ab} \\ \ell\epsilon\pi^{\ell}_{ab}}} \frac{r^{i}_{\ell}(a,b)}{\mu C_i - \lambda_i} = T_{ab} \tag{1.37}$$

In Eq. (1.36) and Eq. (1.37) the summation is over the individual paths π^{ℓ}_{ab} and the set of all paths π_{ab}. The routing variables depend upon the routing assignment and, as shown in Chapter 3, are the quantities which are optimized for certain performance metrics. (Note that the shortest (j,k) route is given by π^{1}_{jk}, the second shortest route by π^{2}_{jk}, and so on.)

Finally, the message routing variables may be used in reformulating Eq. (1.24) for the flow on the ith channel as

$$\lambda_i = \sum_{\substack{j,k \\ i,\ell\epsilon\pi^{\ell}_{jk}}} r^{i}_{\ell}(j,k)\gamma_{jk} \tag{1.38}$$

where the summation is over all the commodities which flow in link i and $r_{\ell}^{i}(j,k)$ is the appropriate routing variable for the (j,k) traffic.

1.5 Conclusions

The results of Section 1.4 pertain to the class of network models for which product form solutions are obtained. The basic assumptions under which these paradigmatic solutions are valid are:

1. Noiseless channels.
2. Infinitesimal nodal processing and propagation delay.
3. Equilibrium conditions.
4. Infinite nodal storage capacity.
5. State-independent routing procedures.
6. Absence of protocol-dependent errors.
7. Independence of interarrival and service time distributions.

The queueing-theoretic formulation presented in this chapter forms the foundation for the research contained in the remaining chapters.

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Chapter 2

NETWORK LEVEL RATE DISTORTION THEORY

2.1 Introduction to Chapter 2

Rate distortion theory has been used in the past to describe the performance of communication channels in which source encoding is followed by data compression and then channel transmission [1,2]. The classical work in rate distortion theory has shown that for a wide class of distortion measures and information sources, there exists a rate distortion function R(D). The importance of R(D) is that it yields a lower bound on the average message distortion, or a lower bound on the required channel capacity. In practice, the theory has not had much impact. The lack of impact may be due to the difficulty in modeling sources or to an operational aversion to data compression schemes [3]. Research in rate distortion theory has centered on calculating R(D) for specific sources and distortion measures, for example [4,5]. The objective of this chapter is to extend single-channel rate distortion theory to computer-communication networks. As a result of this extension, a rate distortion theory for modeling communication errors in networks and for evaluating performance degradation due to channel noise will be constructed.

The motivation for this research stems from noting that single channel communications suffer from bit error rates (BER) ranging from 10^{-3} to 10^{-12} errors/bit. The high rate may be obtained on standard telephone lines operated without error correction, the low rate through the use of conditioned lines, message coding, and error detecting/error correcting protocols [6,7,8]. In general, the penalties for the low bit error rate are found in the additional costs for hardware and/or software to implement the message coding, and in increased channel time delays. In computer-communication networks, the penalty for reliable communications is paid by additional costs for nodal processing, line conditioning, and excessive message delay suffered by the users. These penalties suggest that, although error-free messages are aesthetically pleasing, there are classes of network users who neither require high message fidelity nor are willing to pay for it. As an example, one can imagine a national electronic mail service, in which customers choose levels of service that are functions of message fidelity and message delay. This sort of network tariff plan would have benefits for both the users and the network managers. The user pays for the desired level of service, balancing his requirements against the network tariff schedule. The network managers of such a system will be able to allocate resources based on the desired level of network performance with respect to message time delay, message fidelity, and volume of message traffic.

In this chapter, the network-level rate distortion theory is presented and relationships between message fidelity, time delay, link capacity, and message volume are derived. In Section 2.2, a brief overview of rate distortion theory is presented. In

Section 2.3, the network-level model is formulated along with key definitions. In Section 2.4, the major results for computer-communication networks are derived. Section 2.5 concludes Chapter 2 with a summary of the benefits of the network-level rate distortion theory and a brief recapitulation of key assumptions and results. In Chapter 4, a detailed network example is used to illustrate the conclusions reached in this chapter regarding rate distortion theory.

2.2 A Brief Review of Classical Rate Distortion Theory

The mathematical foundation of rate distortion theory was presented in 1959 by C. E. Shannon [1]. The principle conclusion was "there exists a rate distortion function . . . that measures the 'equivalent rate' of the source for the given level of distortion." Further elaboration of the theory has been concerned with modeling sources, channels, and choosing distortion measures, with excellent summaries of the classical theory contained in [3,8] and especially [2].

The communication system model for which the theory has been developed is shown in Figure 2.1. The discrete time model consists of source, user, channel, source encoder/decoder, and channel encoder/decoder. The source generates a message m(t) which is a member of a set of pre-selected messages. That is, the space of possible source outputs is partitioned into a set of equivalence classes. The source encoder indicates the appropriate equivalence

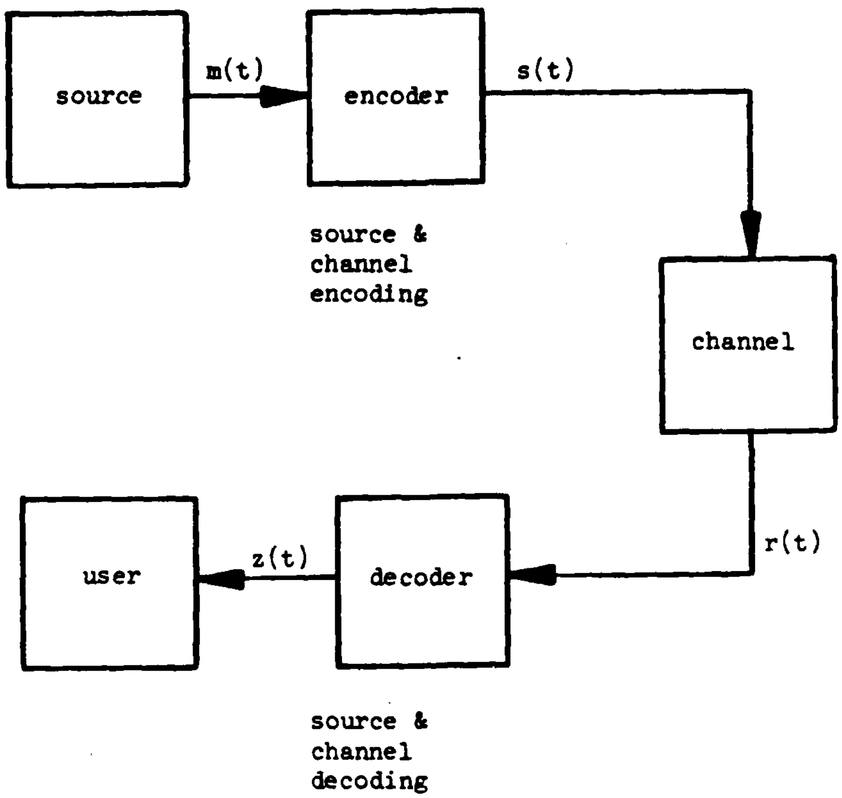

Figure 2.1 Model for classical rate distortion theory

class to the channel encoder. The channel encoder transmits the appropriate message waveform s(t) over the channel.

The channel is the physical link between the source and user. Examples of channels are telephone lines, radio propagation paths, and fiber optic cables. For the purposes of this model, channels of equal capacity and identical distortion properties are considered equivalent. On the user side of the channel, the [possibly] distorted message r(t) is presented to the channel and source decoder. The decoder makes decisions regarding which equivalence class was transmitted and presents the estimate z(t) of the equivalence class to the user.

The mathematical description of this model and the definition of the rate distortion function R(D) are derived from the fundamental concepts of information theory. (These fundamental concepts are concisely presented in [9].)

To quantify this concept in information-theoretic terms, assume a source alphabet M whose members are the letters $\{m(j)\}$ $j = 1,2,\ldots M$ and a user alphabet Z whose members are the letters $\{z(k)\}$ $k = 1,2,3,\ldots Z$. The joint probability P_{jk} is defined on the cartesian product space $MZ = (m(j),z(k))$ where $m(j) = j$, $m(j)\varepsilon M$; $z(k) = k$, $z(k)\varepsilon Z$.

Functions may be defined on the ensemble of alphabets and the joint probability distribution. The random variables of interest are defined via the relations

$$\Pr[m(j) = j] = P_j \tag{2.1}$$

$$\Pr[z(k) = k] = Q_k \tag{2.2}$$

$$\Pr[m(j) = j \text{ and } z(k) = k] = P_{jk} \tag{2.3}$$

The marginal and conditional probabilities are related in the usual way,

$$P_j = \sum_{k=1}^{M} P_{jk} \tag{2.4}$$

$$Q_k = \sum_{j=1}^{M} P_{jk} \tag{2.5}$$

$$P_{j|k} = P_{jk}/Q_k \tag{2.6}$$

$$Q_{k|j} = P_{jk}/P_j \tag{2.7}$$

where $P_{j|k}$ is shorthand for $P(j|k)$, the probability that $m(j) = j$ given that $z(k) = k$.

The simplest and perhaps most useful definition of the rate distortion function assumes the information source to be a discrete memoryless source (D.M.S.). Consider the successive letters generated by the source denoted by the vector

$$\vec{m} = (m_1, m_2, m_3 \ldots . m_n)$$

where m_n is the source letter generated at time t_n.

For the D.M.S., it is assumed that the successive letters are independent, identically distributed, discrete, random variables. The D.M.S. model exhibits two properties:

1. The source output is stationary

2. $P(\vec{m}) = \prod_{t=1}^{n} P(m_t)$

Thus, the source can be completely described as an ensemble $\{m_t, P\}$ where m_t is a random variable indexed by time and P is the corresponding probability distribution. The D.M.S. produces the output $\vec{m}$ which is encoded and sent over the channel to be decoded by the user. The source message $\vec{m}$ is decoded and presented to the user as a selected word $\vec{z}$. Associated with the decoding of $\vec{m}$ into $\vec{z}$ (where possibly $\vec{m} = \vec{z}$) is a cost function or distortion measure $d_n(\vec{m},\vec{z})$ indexed by n, the length of the message $\vec{m}$. Following the definition in [2], the sequence of distortion measures

$$F = \{d_n(\vec{m},\vec{z}),\ 1 \leq n < \infty\} \tag{2.8}$$

is called the fidelity criterion.

A special class of fidelity criteria which has been identified as being computationally tractable is the single letter fidelity criterion. The single letter criterion is the average penalty incurred on a letter by letter basis, i.e.

$$d_n(\vec{m},\vec{z}) = \frac{1}{n} \sum_{t=1}^{n} d(m_t, z_t) \tag{2.9}$$

where $d(m_t, z_t)$ is the single letter distortion measure.

Using the previously defined source and user alphabets and associated probability distributions, the average distortion between a source-user pair can be written as

$$\bar{D} = \sum_{j,k} P_j Q_{k|j} d_{jk} \tag{2.10}$$

where d_{jk} is the penalty for presenting to the user the letter z(k) where the source selected the letter m(j) and the sum is over all $m(j) \varepsilon M$, $z(k) \varepsilon Z$. The conditional transition probabilities $Q_{k|j}$ are called D-admissible if $\bar{D} \leq D$, for some fixed D. The set of D-admissible transition probabilities is set obtained from the rule

$$Q_D = \{Q_{k|j}: \bar{D} \leq D\} \tag{2.11}$$

The average mutual information is defined in the usual way as

$$I(\vec{m},\vec{z}) = \sum_{j,k} P_j Q_{k|j} \log \left(\frac{Q_{k|j}}{Q_k} \right) \tag{2.12}$$

For fixed D, the rate distortion function is then defined by the relation,

$$R(D) = \min_{Q \varepsilon Q_D} I(\vec{m},\vec{z}) \tag{2.13}$$

That is, R(D) is the solution to a convex optimization problem in which the transition probabilities are varied and the solution is subject to linear equality and inequality constraints. Theoretical research in this area has yielded a large number of theorems describing the properties of R(D) under various conditions. Most of these theorems are not necessary for the development of this thesis, so the interested reader is referred to the references, especially [1,2,9]. However, theorems or results required here will be developed as needed.

The significant feature of R(D), as defined here, is that it depends only on the source message statistics and the distortion measure. While this definition of R(D) has been the mathematical basis for data compression, in Section 2.3 the rate distortion concept will be reinterpreted and applied to computer-communication networks.

2.3 Rate Distortion Theory and Computer-Communication Networks

In this section, the network problem is formulated in information-theoretic terms, the rate distortion function for networks is defined, and suggestions for suitable distortion measures are given.

2.3.1 Problem Formulation

As shown in Chapter 3 of this work, the flow in the ith link of a computer-communication network can be written as

$$\sum_{\substack{j,k_\ell \\ i,\ell\in\pi_{jk}}} r_\ell^i(j,k)\gamma_{jk} = \lambda_i \tag{2.14}$$

where the sum is over all commodities which flow in link i, $r_\ell(j,k)$ is the appropriate routing variable for the (j,k) traffic, and

$$0 < r_\ell^i(j,k) \leq 1 \tag{2.15}$$

for all j,k; also, λ_i is the total flow on the ith link in messages/sec., and γ_{jk} is the average rate in messages/sec at which traffic enters the network at node j destined for node k, $j \neq k$.

The network arrival rates are assumed to be independent and Poisson. This assumption is equivalent to assuming that the network message sources are each D.M.S. with respect to message generation.

In terms of a single channel communication model, the multiple commodities on link i are viewed as being transmitted from several sources (one source per commodity), and independently encoded and placed on the channel. The single channel embedded in a network raises questions regarding the channel-network interactions. From the network perspective, there are three logical levels to which information-theoretic questions regarding rate distortion functions and information capacities should be addressed:

1. The link level--What are the effects of multicommodity flow on a single channel?
2. The (j,k) path level--What are the consequences of having source-user (source-destination) paths consisting of several links?
3. The network level--Can basic sufficient and/or necessary conditions be established at the network level for the development of a network view of distortion and capacity?

The following paragraphs address the questions pertinent to these three logical levels.

2.3.2 Link Rate Distortion Function

As indicated in Chapter 1, the building blocks of computer-communication networks are processing nodes and communication links connecting these nodes. It is logical, then, to start the investigation of network rate distortion characteristics by examining the link rate distortion function. This examination begins by defining distortion measures for message and packet switched networks and then examining the properties of rate distortion theory as they relate to multicommodity flow on a single channel.

2.3.2.1 Link Rate Distortion Measures

The classical theory of rate distortion invokes a single-letter fidelity criterion, i.e., the reproduced letter is compared with the source message letter on a letter-by-letter basis. A more suitable measure for computer-communication networks would compare source and distination messages and/or packets, possibly including the message or packet context. For this study, eight distortion measures are introduced in this section. The proposed distortion measures are:

1. Hamming Distance (binary level)

Suppose that the received packet or message differs from the transmitted vector in exactly d_m positions. The number d_m is then said to be the Hamming distance between the transmitted and received messages [10]. On the binary level, assume the source message or packet was sent as $m_t \epsilon \{0,1\}$ for all t, and the received message was $z_t \epsilon \{0,1\}$ for all t. The conditional probability of receiving the message $\vec{z}$ given that the message $\vec{m}$ was sent, assuming that the probability that $m_n = z_n$ is $(1 - p)$ while that for $m_n \neq z_n$ is p, and that there are d_m errors, is

$$P_n(\vec{z}|\vec{m}) = \prod_{t=1}^{n} p(z_t|m_t) = p^{d_m}(1 - p)^{n-d_m} \tag{2.16}$$

where as before n is the message length, p is the bit error rate, and the message source is D.M.S. For the binary case, the Hamming distance may be related to the Hamming weight. If modulo-2 addition of binary symbols is denoted by $\oplus$, and if the Hamming weight $W(\vec{z})$ is defined as the number of 1's in a binary vector $\vec{z}$, then,

$$d_m = W(\vec{z} \oplus \vec{m}) \tag{2.17}$$

Using the Hamming weight, a network level distortion measure $d(\vec{m},\vec{z})$ may be introduced. The Hamming distance distortion measure is defined to be

$$d(\vec{m},\vec{z}) = \frac{1}{n} d_m \tag{2.18}$$

2. Prioritized Hamming Distance (binary level)

In computer-communication networks, packets and messages have varying degrees of importance to the users and to the network managers. Control traffic is essential to maintain network functions, while users may impose their own level of priority on message traffic (and pay for those priorities). This suggests that the Hamming distance distortion measure should be indexed by priority. For this case, the distortion measure is taken to be a linear function of priority and the binary level Hamming distance. The distortion measure for the transmitted message $\vec{m}$ and received message $\vec{z}$ where $\vec{m}$ has priority P is taken to be

$$d(\vec{m},\vec{z}) = \frac{1}{n} d_m + P \tag{2.19}$$

where $d_m = W(\vec{z} \oplus \vec{m})$ and n is the message length. Other forms of distance measures with non-linear dependencies on priority and Hamming distance suggest themselves and are topics of interest for future research.

3. Hamming Distance (character level)

Following the same rationale as in the development of a bit level Hamming distance, suppose that the received message $\vec{z}$ differs from the transmitted message $\vec{m}$ in d_m^c characters. The distortion measure is then

$$d(\vec{m},\vec{z}) = \frac{1}{k} d_m^c \tag{2.20}$$

where k is the message length in characters.

The implications of using this distortion measure are related to the differences in burst error rates (sequences of errors) and bit error rates. Since message characters are composed of strings of n bits where n is typically 5, 8 or 16 bits, burst errors will contribute to increasing the bit error rate directly, but will not have the same effect on the character error rate. For example, suppose the vector $\vec{m}$ was (11011001) where the first four bits comprise one character and the second four bits comprise another. For the purpose of the example, let the received vector $\vec{z}$ be (11011010). The number of bit errors is 2 while the number of character errors is 1. This example, with twice as many bit errors as character errors, suggests that a message with a number of bit errors may still be quite acceptable to the user because the number of character errors is small.

4. Prioritized Hamming Distance (character level)

For the same reason given in the description of the Prioritized Hamming Distance (binary level), a character level, prioritized distortion measure may be appropriate. The measure introduced here is

$$d(\vec{m},\vec{z}) = \frac{1}{k}\, d_m^c + P \tag{2.21}$$

where P is the priority level, d_m^c is the character level Hamming distance, and k is the message length.

The remarks pertaining to other forms of the functional dependence for a prioritized distortion measure are also applicable here. The remaining three distortion measures are formed by normalizing the average of the previously defined metrics. Let D_N^i denote the normalized average measure, then the relationship between the normalized and unnormalized metric is

$$D_N^i = \frac{D^i}{\max\,[D^i]} \tag{2.22}$$

where D^i is the average distortion. Thus the range on each normalized measure is the interval (0,1). The purpose of normalization is to map all measures onto the interval (0,1) and thus provide a standard to facilitate the comparison of different metrics and to simplify the implementation of such a network design feature.

The distortion measures introduced in this section do not have the property of additivity. Additivity is the property, given a user-source pair, a distortion measure, and a single priority message stream $\vec{m}$, say of messages $\vec{m}_1$ and $\vec{m}_2$, then the total distortion of the message stream $\vec{m} = \vec{m}_1 + \vec{m}_2$ is equal to the sum of the distortions of the individual messages

$$d(\vec{m},\vec{z}) = d(\vec{m}_1,\vec{z}_1) + d(\vec{m}_2,\vec{z}_2) \tag{2.23}$$

where the reproduced message stream $\vec{z}$ is equal to the sum of the reproduced individual messages $\vec{z}_1$ and $\vec{z}_2$

$$\vec{z} = \vec{z}_1 + \vec{z}_2 \tag{2.24}$$

2.3.2.2 Properties of the Link Rate Distortion Function

As noted in Section 3.1, the ith link flow is composed of the flows of various commodities

$$\sum r_{\ell}^{i}(j,k)\gamma_{jk} = \lambda_i \tag{2.25}$$

where the sum is over all commodities which flow in link i, $r_{\ell}^{i}(j,k)$ is the routing variable for the (j,k) traffic and

$$0 < r_{\ell}^{i}(j,k) \leq 1 \tag{2.26}$$

In this problem formulation, the traffic statistics γ_{jk} are known to be Poisson and independent. The question then is, what are the properties of the rate distortion function for the ith link?

To begin the investigation, the communications model for the ith link is shown in Figure 2.2. The model consists of a group of subsources representing the sum $\sum r_{\ell}^{i}(j,k)\gamma_{jk}$. Each subsource is a Poisson message generator with average intensity $r_{\ell}^{i}(j,k)\gamma_{jk}$. The subsources are assumed to be independent. The group of subsources merge their messages into a common message stream which is also Poisson with average intensity λ_i. An alternative formulation of this problem would be to view the communication process as a two-person statistical game [11]. In such a formulation, message source statistics are assumed to be entirely unknown and the results are dependent on the rules of the game [12].

Since the characteristics of the traffic requirements matrix $[\gamma_{jk}]$ are known, a direct approach may be used in evaluating the mathematical properties of the ith network link, as opposed to the game-theoretic approach.

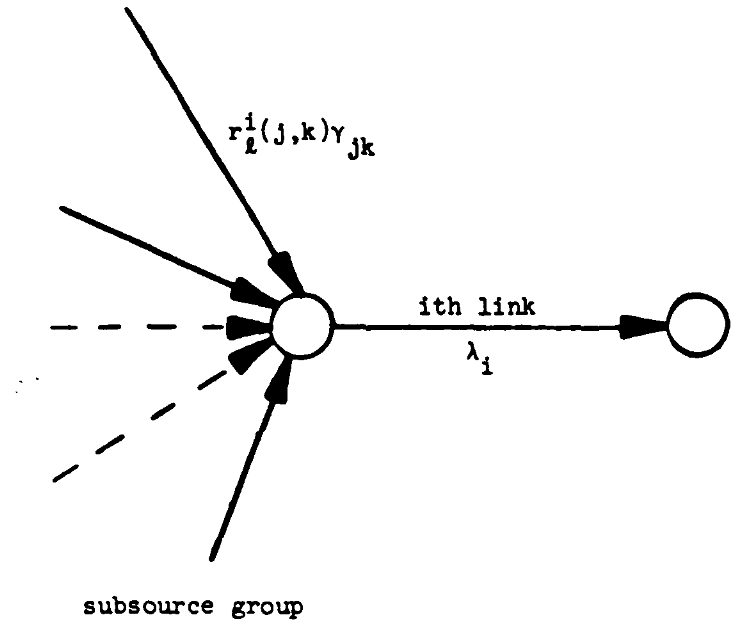

Figure 2.2 Model for multicommodity flow on ith link

First, the properties of the rate distortion function for the ith link (denoted by $R^i(D^i)$) are derived for a single commodity. Then, extensions are made to find the properties of $R^i(D^i)$ for multicommodity flow where these properties are different from those of the single commodity. For the case of a single-commodity in the ith link, the intensity λ_i is

$$\lambda_i = r_\ell^i(j,k)\gamma_{jk} \tag{2.27}$$

and is simply comprised of messages entering the network at node j and destined for node k, $j \neq k$. The probability that the source at node j transmits message $\vec{m}_s$ is defined to be

$$\Pr[\vec{m}_s \text{ transmitted at node j destined for node k}] \equiv P_s^j(j,k) \tag{2.28}$$

The probability that the message $\vec{m}_s$, a member of the (j,k) message set, is transmitted on the ith link is defined to be

$$\Pr[\vec{m}_s \text{ transmitted an ith link}] = P_s^i(j,k) \tag{2.29}$$

In the absence of distortion and for a single commodity flow on the ith link

$$P_s^i(j,k) = r_\ell^i(j,k) \tag{2.30}$$

The transition probability that the message $\vec{z}_t$ is received on the ith link conditioned on the event that the message $\vec{m}_s$ was transmitted, is denoted by,

$$\Pr[\vec{z}_t \text{ received on ith link given that message } \vec{m}_s \text{ was transmitted}]$$

$$= Q_{t|s}^i\,(j,k) \tag{2.31}$$

The notation (j,k) is necessary to denote the commodity being specified on the ith link. Figure 2.3 is a schematic of the relationship between the transmitted and received messages on the ith link.

The average distortion for the ith link for the single (j,k) commodity is

$$\bar{D}^i = \sum_{s,t} P_s^i(j,k) Q_{t|s}^i(j,k) d^i(\vec{m}_s, \vec{z}_t) \tag{2.32}$$

where $d^i(\vec{m}_s, \vec{z}_t)$ is the distortion measure for the ith link.

As in the classical theory, the transition probabilities $\{Q_{t|s}^i(j,k)\}$ are called D^i-admissable if $\bar{D}^i \leq D^i$. The set $Q_D^i(j,k)$ of D^i-admissable transition probabilities on link i for j,k traffic is defined by

$$Q_D^i(j,k) = \{Q_{t|s}^i(j,k) : \bar{D}^i \leq D^i\} \tag{2.33}$$

The average mutual information on the ith link for a single (j,k) commodity is

$$I_{jk}^i(\vec{m},\vec{z}) = \sum_{s,t} P_s^i(j,k) Q_{t|s}^i(j,k) \log \left[\frac{Q_{t|s}^i(j,k)}{Q_t^i(j,k)} \right] \tag{2.34}$$

where the sum is over the set of all possible input messages $\{\vec{m}_s\}$ and all possible output messages $\{\vec{z}_t\}$. The rate distortion function on the ith link for the (j,k) commodity is

$$R_{jk}^i(D^i) = \min_{Q_D^i(j,k)} I_{jk}^i(\vec{m},\vec{z}) \tag{2.35}$$

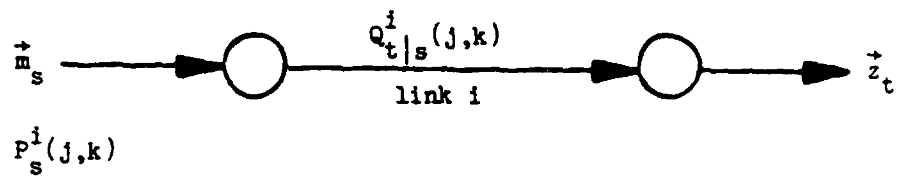

Figure 2.3 Relationships between transmitted and received messages on link i

For a single commodity, $R^i_{jk}(D^i)$ is the measure of interest on the ith link. For a multicommodity flow, the presence of additional commodities must be considered in the calculation of the link rate distortion function.

The average mutual information on the ith link for all commodities is dependent on the joint probability functions for all commodities. For example, the input message probabilities for message $\vec{m}_s$ and all traffic $j_1, j_2, j_3 \ldots k_1, k_2, k_3, \ldots$, which can be abbreviated as $(\vec{j},\vec{k})$, is denoted by $P^i_s(\vec{j},\vec{k})$ and similarly for the transition probabilities.

The general link average mutual information is

$$I^i(m,z) = \sum_{\substack{s,t\\ \vec{j},\vec{k}}} P^i_s(j,k) Q^i_{t|s}(\vec{j},\vec{k}) \log \left[\frac{Q^i_{t|s}(\vec{j},\vec{k})}{Q^i_t(\vec{j},\vec{k})} \right] \qquad (2.36)$$

where the sum is over the cartesian product spaces of all possible input/output message pairs, and s and t may be thought of as n-tuples formed by the union of all input message sets and output message sets, respectively. The message set $\{\vec{m}_s\}$ can be written in terms of the commodities comprising it, i.e.,

$$\{\vec{m}_s\} = \{\vec{m}_{s_1}\} \cup \{\vec{m}_{s_2}\} \cup \{\vec{m}_{s_3}\} \cup \ldots$$

where $s_1, s_2 \ldots$ are the indices for the constituent commodities (j_1,k_1), $(j_2,k_2) \ldots$. The message set $\{z_t\}$ is formed in an analogous fashion.

Because of the source independence, the general expression may be further simplified. Noting that source independence implies that message probabilities factor, one has that

$$P_s^i(\vec{j},\vec{k}) = P_{s_1}^i(j_1,k_1)P_{s_2}^i(j_2,k_2)P_{s_3}^i(j_3,k_3)\ldots. \qquad (2.37)$$

The average mutual information then simplifies to

$$I^i(\vec{m},\vec{z}) = \sum_{j,k}\sum_{s,t} P_s^i(j,k)Q_{t|s}^i(j,k)\log\left[\frac{Q_{t|s}^i(j,k)}{Q_t^i(j,k)}\right] \qquad (2.38a)$$

$$= \sum_{j,k} I_{jk}^i(\vec{m},\vec{z}) \qquad (2.38b)$$

The summation is over all possible input message/output message pairs for each commodity flowing on the ith link.

Then for fixed D^i, the rate distortion function for the ith link is defined as

$$R^i(D^i) = \min_{Q^i \varepsilon Q_D^i} I^i(\vec{m},\vec{z}) \qquad (2.39)$$

where

$$Q_D^i = \{Q_{t|s}^i : \bar{D}^i \leq D^i\}$$

The solution for $R^i(D^i)$ is thus a convex mathematical programming problem. Specifically, the average mutual information $I^i(\vec{m},\vec{z})$ is a convex function of a vector variable (which is also convex) and must be minimized subject to the linear equality and inequality constraints

$$Q_{t|s}^i(j,k) \geq 0 \qquad (2.40a)$$

$$\sum_t Q_{t|s}^i(j,k) = 1 \qquad (2.40b)$$

$$\sum_{\substack{j,k\\s,t}} P_s^i(j,k)Q_{t|s}^i(j,k)d^i(\vec{m}_s,\vec{z}_t) = \bar{D}^i \tag{2.41}$$

In the example of Chapter 4, a link rate distortion function will be calculated and applied to a network.

Under very general conditions, the link rate distortion function (RDF) is decomposable into the individual commodity rate distortion functions on the link, as the following theorem shows.

RDF Decomposition Theorem

Assume that two commodities flow on link i with

$$\lambda_i = \gamma^a + \gamma^b$$

where a, b are the commodities, and a, b are independent, each with its own distortion measure $d^{ia}(\vec{m}_s,\vec{z}_t)$, $d^{ib}(\vec{m}_{s'},\vec{z}_{t'})$ and individual distortion functions $R^{ia}(D^{ia})$ and $R^{ib}(D^{ib})$ respectively. Then,

$$R^i(D^i) = R^{ia}(D^{ia}) + R^{ib}(D^{ib}) \tag{2.42}$$

and

$$D^i = D^{ia} + D^{ib}$$

Proof. The transition probabilities for the two independent commodities γ^a and γ^b are denoted as $Q_{t|s}^{ia}$ and $Q_{t'|s'}^{ib}$. The transition probability for $R^i(D^i)$ is denoted by $Q_{tt'|ss'}$.

First, it is shown that for any minimal assignment $\{Q_{tt'|ss'}^i\}$, the product $\{Q_{t|s}^{ia}Q_{t'|s'}^{ib}\}$ gives the same distortion.

Choose the $Q^{ia}_{t|s}$ and $Q^{ib}_{t'|s'}$ such that

$$Q^{ia}_{t|s} = \sum_{t',s'} P^{ia}_{s'} Q^{i}_{tt'|ss'} \tag{2.43a}$$

$$Q^{ib}_{t's'} = \sum_{s,t} P^{ib}_{s} Q^{i}_{tt'|ss'} \tag{2.43b}$$

where $P^{ia}_{s'}$, $P^{ib}_{s'}$ are the input message probabilities for the two commodities on link i. Then, $Q^{ia}_{t|s}$, $Q^{ib}_{t'|s'}$ are clearly satisfactory probabilities and give the same total distortion as the product assignment as seen in the following argument. First, the distortion given by the product is

$$\sum_{ss',tt'} P^{ia}_{s} P^{ib}_{s'} Q^{ia}_{t|s} Q^{ib}_{t'|s'} [d^{ia}(\vec{m}_s,\vec{z}_t) + d^{ib}(\vec{m}_{s'},\vec{z}_{t'})]$$

$$= \sum_{s,t} P^{ia}_{s} Q^{ia}_{t|s} d^{ia}(m_s,z_t) + \sum_{s',t'} P^{ib}_{s'} Q^{ib}_{t'|s'} d^{ib}(\vec{m}_s,\vec{z}_{t'}) \tag{2.44}$$

which is also the distortion using $Q^{i}_{tt'|ss'}$.

Second, the choice $Q^{ia}_{t|s}, Q^{ib}_{t'|s'}$ is less than or equal to the minimum given by the joint probability, $Q^{i}_{tt'|ss'}$.

Using the entropy function in the usual way,

$$\begin{aligned} R^i(D^i) &= H(s,s') - H(s,s'|t,t') \\ &\geq H(s,s') - H(s|t) - H(s'|t') \\ &= H(s,s') - \hat{H}(s|t) - \hat{H}(s'|t') \end{aligned} \tag{2.45}$$

where $\hat{H}()$ denotes an entropy with the product assignment $Q^{ia}_{t|s} Q^{ib}_{t'|s'}$. Since the commodities γ^a, γ^b are independent, then

$$H(s,s') = H(s) + H(s') = \hat{H}(s) + \hat{H}(s') \tag{2.46}$$

and

$$R^i(D^i) \geq R^{ia}(Q^{ia}) + R^{ib}(Q^{ib}) \tag{2.47}$$

Since $R^i(D^i)$ is the minimum over the convex space Q_D^i, it follows immediately that

$$R^i(D^i) = R^{ia}(D^{ia}) + R^{ib}(D^{ib}) \quad \text{Q. E. D.} \tag{2.48}$$

The importance of this theorem is that it allows the network designer to evaluate the rate distortion on a commodity by commodity basis and then design individual network links to maintain a prescribed link-level of distortion.

2.3.2.3 <u>A Link Capacity Measure</u>

Intrinsic to the development of rate distortion functions for computer networks is a usable definition of network capacity. The foundation of that definition is the link capacity. The link capacity $\tilde{C}_i$ in bits per message per channel usage is defined as

$$\tilde{C}_i = \max_{\{P_s^i\}} I^i(\vec{m},\vec{z}) \tag{2.49}$$

That is, the link capacity for a discrete memoryless channel is the maximum of the average mutual information with respect to all possible input message probabilities. To make $\tilde{C}_i$ a useful measure requires knowledge of T_c, the time period for each message transmission. When T_c is known, then the link capacity is

$$C_i = \tilde{C}_i/T_c \text{ bits/sec-msg} \tag{2.50}$$

In practice, C_i is known a priori for a particular link and calculations proceed from there. In Chapter 4, a link capacity is calculated by maximizing the average mutual information.

2.3.3 Routing Level RDF

Given that the RDF can now be calculated for a specific link or for a single commodity on a specific link, what may be inferred regarding the (j,k) traffic following a path π_{jk} or paths π^{ℓ}_{jk}? Single path routing and alternate routing are considered in the next two subsections.

2.3.3.1 Static Routing (Single-Path Routing)

Assume that commodity (j,k) is routed over n links comprising path π_{jk}. On the first link, the (j,k) commodity will have the RDF $R^1_{jk}(D^1)$

$$R^1_{jk}(D^1) = \min_{Q^1_D(j,k)} I^1_{jk}(\vec{m},\vec{z}) \tag{2.51}$$

Where the average mutual information $I^1_{jk}(\vec{m},\vec{z})$ is

$$I^1_{jk}(\vec{m},\vec{z}) = \sum_{s,t} P^1_s(j,k)Q^1_{t|s}(j,k) \log \left[\frac{Q^1_{t|s}(j,k)}{Q^1_t(j,k)} \right] \tag{2.52}$$

The notation indicates this is the average mutual information on the first link of the path π_{jk} for the (j,k) commodity. The expression for the second link is similar,

$$I^2_{jk}(\vec{m},\vec{z}) = \sum_{s,t} P^2_s(j,k)Q^2_{t|s}(j,k) \log \left[\frac{Q^1_{t|s}(j,k)}{Q^1_t(j,k)} \right] \quad (2.53)$$

In this model, the output of the second link depends statistically only on the input to this link, so that in the expression for the second link, the input probabilities $P^2_s(j,k)$ then depend on the transition probabilities of the first link and the message input probabilities of the first link through the relation

$$P^2_s(j,k) = \sum_{a_1} P^1_{a_1}(j,k)Q^1_{s|a_1}(j,k) \quad (2.54)$$

In evaluating $R^1_{jk}(D^1)$ the set of transition probabilities $\{Q^1_{s|a}\}$ were selected to minimize $I^1_{jk}(\vec{m},\vec{z})$ and are thus known in the above equation for $P^2_2(j,k)$. $R^2_{jk}(D^2)$ is

$$R^2_{jk}(D^2) = \min_{Q^2_D(j,k)} I^2_{jk}(\vec{m},\vec{z}) \quad (2.55)$$

The average mutual information for the nth link is then

$$I^n_{jk}(\vec{m},\vec{z}) = \sum_{s,t} P^n_s(j,k)Q^n_{t|s}(j,k) \log \frac{Q^n_{t|s}(j,k)}{Q^n_t(j,k)} \quad (2.56)$$

where the nth line message input probabilities can be related to the source message probabilities and the link transition probabilities on the path π_{jk} by the relation

$$P_s^n(j,k) = \sum_{a_1,a_2\ldots a_{n-1}} P_{a_1}^1 \left(\prod_{i=1}^{n-2} Q^i_{a_{i+1}|a_i}(j,k) \right) Q^{n-1}_{s|a_{n-1}}(j,k) \quad (2.57)$$

If the link transition probabilities are identical over the path π_{jk}, i.e., if

$$Q_{s|t}(j,k) = Q^{\ell}_{s|t}(j,k)$$

for all ℓ, s, and t, the expression for the input probabilities can be expressed as a homogenous discrete-time Markov chain where the input probabilities $P_s^1(j,k)$ represent initial state probabilities. Specifically, let $\vec{P}^{\ell}(j,k)$ be the column vector

$$\vec{P}^{\ell}(j,k) = \begin{bmatrix} P_1^{\ell}(j,k) \\ \cdot \\ \cdot \\ \cdot \\ P_s^{\ell}(j,k) \end{bmatrix} \quad (2.58)$$

where the entries are the message probabilities for the ℓth link, and define the transition matrix $\underline{Q}(j,k)$ for the path π_{jk} to be

$$\underline{Q}(j,k) = [Q_{m|n}(j,k)] \quad (2.59)$$

where the element $Q_{m|n}(j,k)$ is in the mth row and nth column. Then, the result for the ℓth link input probabilities is

$$\vec{P}^{\ell}(j,k) = [\underline{Q}(j,k)]^{\ell}\vec{P}^1(j,k) \quad (2.60)$$

where $[\underline{Q}(j,k)]^{\ell}$ is the ℓth power of the transition matrix and $\vec{P}^1(j,k)$ is the vector of input message probabilities for the first link.

Here, one must distinguish between the transition probabilities $Q^{\ell}_{m|n}(j,k)$ which determine the input message probabilities in successive links and the set $Q_D(j,k)$ which is varied subject to maintaining the average distortion. The former transition probabilities are determined by the link characteristics, while the latter is the set of transition probabilities over which the mutual information is varied until the greatest lower bound is obtained.

The rate distortion measure for the single commodity with static routing ((j,k) traffic) is defined to be

$$R_{jk}(D) = \frac{1}{n} \sum_{\ell \in \pi_{jk}} R^{\ell}_{jk}(D^{\ell}) \qquad (2.61)$$

and

$$D = \sum_{\ell \in \pi_{jk}} D^{\ell}$$

where the summation is over the n links which comprise path π_{jk}. (If additional clarity is needed, the rotation D_{jk} and D^{ℓ}_{jk} can be used for D and D^{ℓ}, respectively.)

Unfortunately $R_{jk}(D)$ is not a useful measure in general. However, for certain cases, e.g., equiprobable input message distributions, the (j,k) traffic level RDF is readily calculable and is important in characterizing network performance, as will be shown in Chapter 4. Additional remarks and results concerning $R_{jk}(D)$ are also presented in Chapter 4 along with a computational example.

2.3.2.2 Alternate Routing

In the case of message streams being split between source and destination, as in the minimum hop algorithms proposed in Chapter 3, the traffic intensity for a given commodity is scaled by the routing variable on the ith link,

$$r_\ell^i(j,k)\gamma_{jk} = j\text{-kth component of } \lambda_i$$

The message input probabilities at the source are not affected by alternate routing and the input probabilities for each routing are determined as in the static case. The average measure for alternate routing is an extension of the single routing measure and is defined to be

$$R_{jk}(D) = \frac{1}{m} \sum_{\substack{m \\ m\epsilon\pi_{jk}^m}} \frac{1}{n_m} \sum_{\substack{\ell \\ \ell\epsilon\pi_{jk}^m}} R_{jk}^\ell(D^\ell) \tag{2.62}$$

and

$$D = \sum_{\ell\epsilon\pi_{jk}^\ell} D^\ell$$

where the summations are over all m alternate routings and the n_m links of each of the m alternate paths, where π_{jk}^m denotes the mth alternate path for the (j,k) traffic. The caveats following Eq. (2.61) pertain here also.

2.3.4 Routing Optimization Strategies

Traditionally, the driving design parameter in message and packet routing strategies has been link capacity constraints.

However, by accepting message distortion as a design variable, an additional degree of freedom is added to possible routing strategies in the form of variable link capacities. The implications of this additional degree of freedom require the detailed study which is given in Chapter 3. However, the potential routing strategies are introduced here as part of the logical development of this chapter. The strategies are:

1. Minimization of Path Rate Distortion via Minimum Hop--One practical form for the minimization of message distortion is via the minimization of the average number of links the set of all commodities traverse. This is the network minimum hop routing (a hop occurs each time a link is traversed by a commodity) and minimizes the objective function

$$J = \frac{1}{\gamma} \sum \lambda_i \tag{2.63}$$

This strategy is of greatest interest because of its practicality and is thoroughly examined in Chapter 3.

2. Path Rate Distortion Equalization (PRDE)--The goal of the strategy is to equalize the ratio $C_i/R^i(D^i)$ for all links. The effect is to balance message rates throughout the network.

3. Path Rate Distortion Prioritization (PRDP)--The objective of this policy is to maximize the ratio $C_{jk}/R_{jk}(D)$ for some commodities at the expense of others, i.e. to give a higher priority to certain commodities.

2.3.5 Network Capacity and Relation between Message Delay and Rate Distortion

In previous sections, the network queueing model has been presented as has a network model for representing message errors. One expects that, since the network message traffic is susceptible both to queueing delays and distortion due to noise, a functional relationship can be developed between the information-theoretic results and the queueing-theoretic results. In this section, such a relationship is developed and as a corollary to that work, the concept of single channel capacity is extended to the network level.

In Section 2.3.5.1, the network capacity is introduced and defined. In Section 2.3.5.2, the queueing delay-rate distortion relationships are analyzed and the results are illustrated via an example in Chapter 4.

2.3.5.1 Network Capacity

In Section 2.3.2.3, the link or single channel capacity measure was defined in terms applicable to network usage. These results rely on the single channel message network definition for capacity $\tilde{C}_i$ (or equivalently C_i in units of bits/second).

The network measures are defined here in terms of traffic requirements and network topology. The definitions are formulated in an information-theoretic context with the focus on network applicability. The path capacity C_{jk} for a jk path of n links is defined as

$$C_{jk} \equiv \sum_{\substack{i \\ i\epsilon\pi_{jk}}} C_i \tag{2.64}$$

If alternate routing is used for (j,k) traffic, the definition is obviously

$$C_{jk} \equiv \sum_{\substack{\ell \\ \ell\epsilon\pi_{jk}^{\ell}}} r_{\ell}(j,k) \sum_{\substack{i \\ i\epsilon\pi_{jk}^{\ell}}} C_i \equiv \sum_{\substack{\ell \\ \ell\epsilon\pi_{jk}}} C_{jk}^{\ell} \tag{2.65}$$

where $r_{\ell}(j,k)$ is the routing variable for the ℓth path for the (j,k) traffic and the summations are over all (j,k) traffic paths π_{jk}^{ℓ} and over the links in each π_{jk}^{ℓ}. In the case of static routing, the requirement for finite distortionless message delay leads to the necessary condition

$$C_{jk} > n\Big(\sum_{j,k} \gamma_{jk}\Big)\frac{1}{\mu} \tag{2.66}$$

where n is the number of links in path π_{jk}, $\frac{1}{\mu}$ is the average message length, and the summation is over the commodities flowing on the links defined by π_{jk}.

The corresponding necessary condition for the case of alternate routing is

$$C_{jk} > \sum_{\substack{\ell \\ \ell\epsilon\pi_{jk}^{\ell}}} r_{\ell}(j,k)n^{\ell}\Big(\sum_{\substack{j,k \\ C_i\epsilon\pi_{jk}^{\ell}}} \gamma_{jk}\Big)\frac{1}{\mu} \tag{2.67}$$

where n^{ℓ} is the number of links in path π_{jk}^{ℓ}. Finally, these capacity results can be applied to a generalized network. Let $C_T \equiv$ the capacity of the network. The appropriate definition for total network capacity is

$$C_T = \sum_{i=1}^{M} C_i = \sum_{j,k} C_{jk} \tag{2.68}$$

where there are M links in the network.

For the general case of alternate routing, the necessary condition for finite distortionless message delay becomes

$$C_T > \sum_{j,k} \sum_{\substack{\ell \\ \ell \epsilon \pi^{\ell}_{jk}}} r_\ell(j,k) n^\ell \Big(\sum_{\substack{j,k \\ C_i \epsilon \pi^\ell}} \gamma_{jk} \Big) \frac{1}{\mu} \tag{2.69}$$

The necessary conditions represent a least lower bound on required network capacity for error-free communications.

Of course, the upper bound on network capacity is constrained by cost and available technology.

2.3.5.2 Queueing Delay and Rate Distortion

In section 2.3.5.1, above, a definition of capacity was introduced. The definition characterizes link capacity in units of bits/message-channel usage. Compatible with and related to that capacity definition is the link rate distortion function $R^i(D^i)$ which has the dimensions of bits/source message. The primary result from classical distortion theory which is brought to this development is the change in channel capacity due to the RDF $R^i(D^i)$. The effective link capacity becomes

$$C_{i\ \mathrm{eff}} = \frac{\tilde{C}^i}{T_c R^i(D^i)} \ \frac{\text{messages}}{\text{sec}} \tag{2.70}$$

where the period of a channel usage is T_c and $\tilde{C}^i$ was defined in Section 2.3.2.3.

In practice, T_c may be calculated from fundamental considerations of hardware (modem/terminal/line) and software (protocols/coding) characteristics.

The above equation may be taken to mean that the link capacity is increased by $1/R^i(D^i)$ when messages are transmitted with an average distortion not exceeding D^i. For the broad class of network models in which queueing delay is a function of link capacity and message flow rate, i.e.,

$$T_i = f(\lambda_i, C_i) \tag{2.71}$$

there is an explicit assumption of communications reliability. That is, the communications are assumed to be error-free. With this work, however, the queueing delay becomes a function of distortion D^i, as well as λ_i, and $\tilde{C}_i$. Hence,

$$T_i = f(\lambda_i, C_{ieff}) = f\left(\lambda_i, \frac{C_i}{T_c R^i(D^i)\mu}\right) \tag{2.72}$$

or, in general,

$$T_i = g(\lambda_i, \tilde{C}_i, T_c, R^i(D^i), \mu)$$

where g is a convex function of λ_1. Using the explicit example of Chapter 1, one has

$$T_i(\lambda_i, \mu C_i) = \frac{1}{\mu C_i - \lambda_i} \tag{2.73}$$

and making the necessary changes in notation yields

$$T_i(\lambda_i, C_i, R^i(D^i)) = \frac{R^i(D^i)}{C_i - \lambda_i R^i(D^i)} \tag{2.74}$$

where $\tilde{C}_i/\mu T_c$ has been combined into C_i, the capacity measure in units of messages/sec. For a typical $R^i(D^i)$ as shown in Figure 2.4, T_i has the behavior shown in Figures 2.5 and 2.6. Clearly, T_i remains convex in λ_i for fixed D^i and C_i. For fixed C_i and λ_i, T_i is concave since $R^i(D^i)$ is concave.

For finite message delay, $T_i < \infty$, the message rate is bounded by

$$\lambda_i < C_i/R^i(D^i) \tag{2.75}$$

For fixed C_i, that bound is convex as a function of distortion. This is intuitively satisfying, since here the claim is that the message rate λ_i may be increased if message distortion is allowed and the message delay will still remain finite.

Using the network queueing model from Chapter 1, the average message delay in the network is found to be

$$T = \frac{1}{\gamma} \sum_{i=1}^{M} \frac{\lambda_i R^i(D^i)}{C_i - \lambda_i R^i(D^i)} \tag{2.76}$$

for

$$D^i_{min} \leq D^i \leq D^i_{max}$$

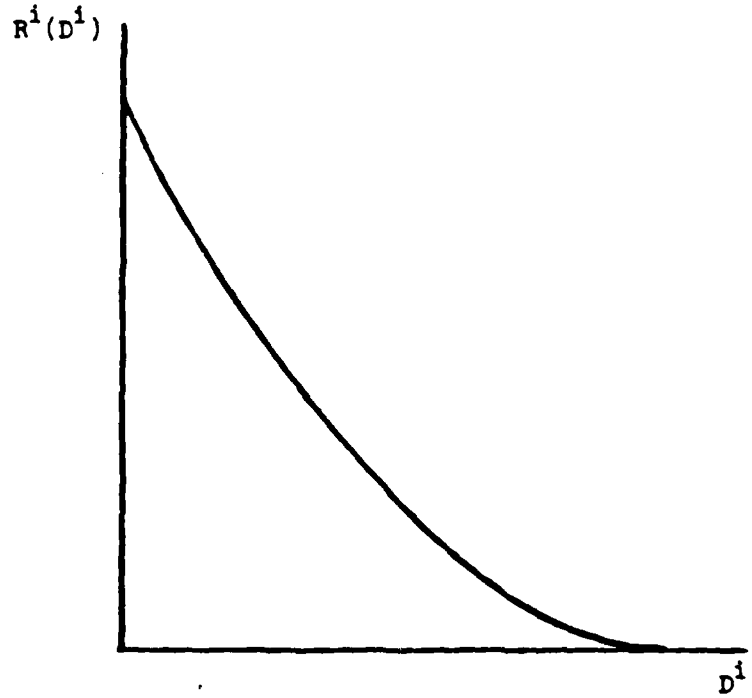

Figure 2.4 A typical link rate distortion function

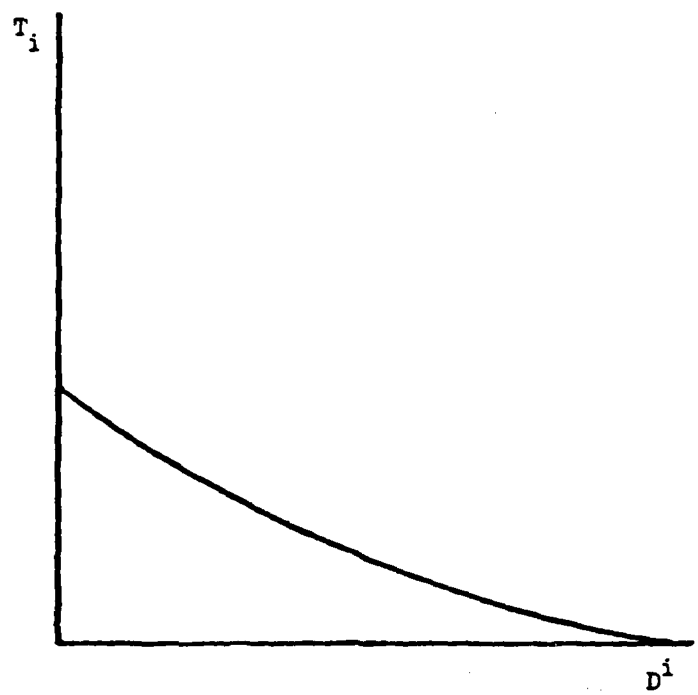

Figure 2.5 The link delay as a function of message distortion

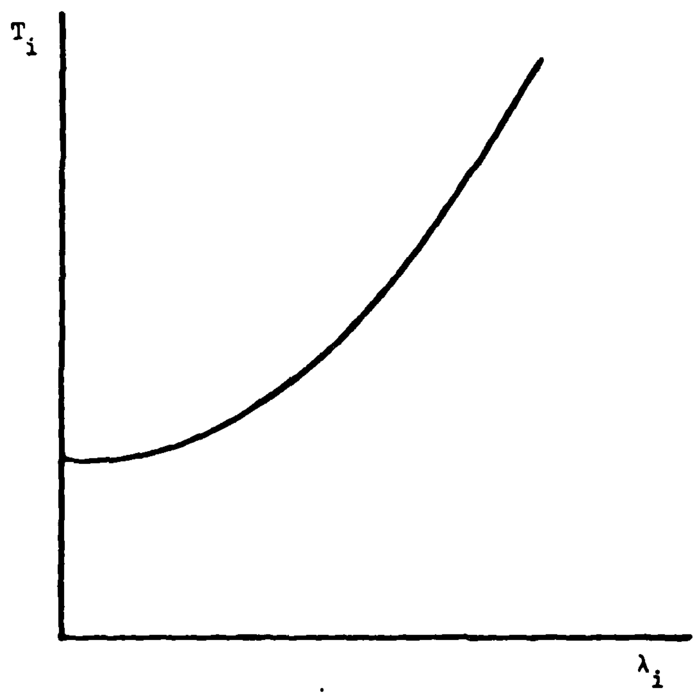

Figure 2.6 The link delay as a function of message rate

In Chapter 4, the interrelationship between message distortion and queueing delay is fully illustrated via a network example.

2.4 Conclusions

The principal result of this chapter is the message distortion--queueing delay relationship developed in Section 2.3. The information-theoretic approach has produced a remarkably simple method for handling network distortion. This general model and its relatively simple methodology should be contrasted with the specialized model in [14] or the unwieldy generating function methodology used in [15,16].

Chapter 2 Notes

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Chapter 3

ROUTING STRATEGIES

3.1 Introduction to Chapter Three

The purpose of this chapter is to examine in detail the routing strategies briefly mentioned in Chapter 2. The focus will be on the Minimum Hop (Min-Hop) policy for two reasons. First, it optimizes the network performance from an overhead traffic and message distortion perspective. Second, it could be implemented in a general large-scale network either as a centralized or decentralized algorithm. This research has developed several methods for calculating and implementing the Min-Hop policy and these developments will be presented in Section 3.2. It should be noted that much of the research included in Section 3.2 has been published or submitted for publication in [1], [2], and [3]. Section 3.3 analyzes the implications of the Path Rate Distortion Equalization (PRDE) policy for network implementation of the PRDE strategy is also presented. Section 3.4 presents the implementation methodology for the network optimization via Path Rate Distortion Prioritization (PRDP). The chapter concludes with Section 3.5, a review of salient features of the three routing strategies.

3.2 The Min-Hop Strategy

The Min-Hop Strategy is to minimize the number of links each commodity traverses, i.e., minimize the number of hops messages make in moving from source to destination. In the following paragraphs the Min-Hop strategy is developed and analyzed in detail and illustrated using network examples. In Section 3.2.1, the rationale for emphasizing the Min-Hop policy and its key position as an optimizer of network performance with respect to message integrity is presented. In Section 3.2.2, the basic optimization problem is presented and a dynamic programming approach to the calculation of the Min-Hop flow assignment is developed and illustrated with a network example. In Section 3.2.3, a centralized Min-Hop routing algorithm constrained by average end-to-end message delay is developed and illustrated with a network example. A decentralized algorithm, in two parts, is developed and presented in Section 3.2.4. The first part of the distributed algorithm yields a Min-Hop flow assignment and the second part modifies the flow assignment to satisfy end-to-end delay constraints.

3.2.1 Rationale for Min-Hop Strategy

The Min-Hop algorithm proposed in this research is designed to avoid store and forward congestion due to certain features found in real-world networks. The features which contribute to thruput degradation and system deadlock are the finite capacity communication lines, the finite capacity buffers of the switch nodes, and the presence of noise on the communication links. These three features are examined in the following paragraphs and the relation to the Min-Hop algorithm is highlighted.

In an actual network, the switching nodes (communications processors) have a finite amount of storage for holding incoming and outgoing packets. The packets reside in what may be considered for modeling purposes as a common buffer pool. When the buffer is full, incoming packets can not be received and are lost to the system or have to be retransmitted thus contributing to channel congestion and consequent performance degradation. To avoid this system condition, various flow control policies have been formulated, the most prominent group of policies being variations of channel queue limit mechanisms. In one common version of the channel queue limit mechanism, the switching node will signal other nodes that a certain percentage of the buffer capacity is occupied and that no further packets will be accepted. This requires potential senders to hold the packets intended for that switching node so that the packets are not lost. This mechanism can result in source-to-destination deadlock. As an illustration of source-to-destination deadlock consider Figure 3.1. An end-to-end path is shown embedded in a general network. The nodes A thru D are connected to other nodes (not shown) in the general network. Source node A wishes to transmit to destination node D via intermediate nodes B and C. Suppose node C has reached its channel queue limit due to heavy traffic involving other commodities. Then node B would hold packets destined for node C. Eventually node B would reach its buffer

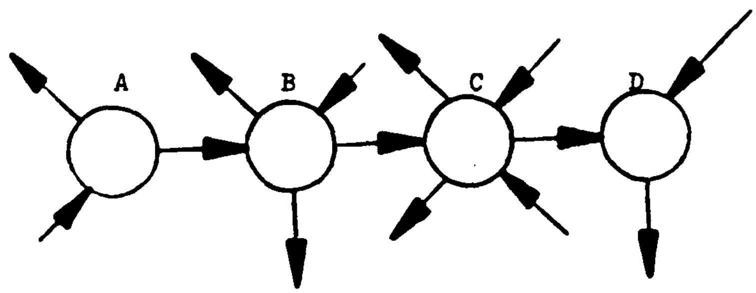

Figure 3.1 Network example demonstrating source-to-destination deadlock

threshold and no longer accept packets. Source node A would not be able to transmit to destination node D, resulting in the form of congestion known as source-to-destination deadlock.

To quantify this concept, suppose the network is well balanced such that line, node, and buffer utilizations are approximately the same. Assume further that regardless of the buffer partitioning policy, the probability of a node reaching its channel queue limit is the same for all nodes, i.e.,

$$\Pr[\text{node i reaches threshold}] = p \tag{3.1}$$

for all nodes. For a path π_{ab} with n links and n + 1 nodes, the probability that a deadlock condition is obtained is,

$$\Pr[\text{source-destination deadlock}] = \sum_{k=1}^{m} \binom{n}{k} p^k (1 - p)^{n-k} \tag{3.2}$$

where $\binom{n}{k}$ is the Bernoulli coefficient. Clearly by minimizing n, which is a feature of the Min-Hop algorithm, the probability of deadlock is minimized.

The second 'real' aspect of networks is the finite line capacity and the associated communication cost. In fact, this tends to be the most serious bottleneck in networks and the most significant cost factor, especially in local area networks. Clearly, by implementing Min-Hop policies the number of communication lines and the total network utilization is minimized as is the communication cost, and the network resource per packet investment. The third 'real' feature which contributes to network performance degradation is communication channel distortion, that is, line noise. A simple "back-of-the envelope" calculation illustrates this problem and the Min-Hop contribution to the solution. Assume a path π_{ab} with n links, each link having a probability of error (bit error rate) of σ. Assume also that the link-level protocol will detect bit errors and require retransmission of the N-bit packets. It is easy to show [4] that the average number of transmissions <T>, required per hop per packet is,

$$\langle T\rangle = \frac{1}{(1-\sigma)^N} > 1 \qquad (3.3)$$

for $\sigma > 0$.

The average number of transmissions required on the path π_{ab} is,

$$n\langle T\rangle = \frac{n}{(1-\sigma)^N} > n \qquad (3.4)$$

Clearly, by minimizing n, the number of message hops, the average number of transmissions to move packets from network sources to network destinations is also minimized, effectively increasing system efficiency and thruput.

As an alternative to using an error detect/correct protocol and requiring message retransmissions, the system could move messages from node to node without checking for errors. In that case, as a message progressed through the network, errors would be accumulated along the message path until it reached its destination. Under the reasonable assumption that the network is balanced in terms of traffic routing and communications integrity, the total number of errors in a received message will be directly proportional to the message path length. Thus, to minimize overall message distortion throughout the network the average message path should be minimized which, of course, is the objective of the Min-Hop strategy.

In the following section, the problem formulation for Min-Hop routing is developed and a method for calculation of the solution, unconstrained by average time delay, is presented.

3.2.2 D.P. Approach to Centralized Min-Hop

A fundamental problem in the design of message-switched telecommunication networks is that of flow assignment and routing [5-11]. Basically, the problem consists of specifying paths for multicommodity flow between all sources and destinations such that some suitable measure of network performance is optimized. This flow assignment is subject to conservation of total average flow at each node, conservation of average flow commodity-by-commodity at each node, and to link capacity constraints. The presentation is as follows. First, the problem formulation and the development of the associated recurrence relation is presented. Then the solution method is illustrated via a six-node, nine-link, four-commodity network in the last paragraph. Consider a message-switched network with N nodes and M directed links in which message lengths are independent and exponentially distributed with mean $1/\mu$ bits, and the network message arrival processes are independent and Poisson. Let γ_{jk} denote the average rate in messages per sec at which traffic enters the network at node j destined for node k, $j \neq k$, and take C_i to be the capacity of link i in bits/sec and λ_i the average total flow on that link in messages/sec. Finally, it is assumed that the interarrival and transmission times on each link are independent of each other. For minimum hop flow assignment, the function to be minimized is

$$J = \frac{1}{\gamma} \sum_{i=1}^{M} \lambda_i \tag{3.5}$$

where

$$\gamma = \sum_{j,k} \gamma_{jk}$$

The objective function is, of course, the average path length throughout the network [10, p. 327] and can be viewed as the sum of all link-wise network traffic or as the sum of all commodity-wise traffic. Consider the latter and let $r_\ell(j,k)$ be the message routing variable as defined in Section 1.4.3. For each (j,k) pair, i=1, ...,I_{jk} where the value of I_{jk} is limited by the network topology, but may be selected to be less than this limit by the designer. Nonnegativity of flow and conservation of flow require that

$$r_\ell(j,k) \geq 0 \tag{3.6a}$$

and

$$\sum_\ell r_\ell(j,k) = 1 \tag{3.6b}$$

respectively, for all ℓ,j,k in Eq. (3.6a) and for each (j,k) pair in Eq. (3.6b). Letting n_{jk}^{ℓ} be the number of links in the ℓth shortest (j,k) path, rewrite Eq. (3.5) as

$$J = \frac{1}{\gamma} \sum_{\ell,j,k} \gamma_{jk} r_{\ell}(j,k) n_{jk}^{\ell} \qquad (\ .7)$$

where the sum over ℓ includes all possible paths for each (j,k) commodity. The minimization of J in Eq. (3.5) is subject to link capacity constraints which are specified by

$$0 \leq \lambda_i \leq \alpha_i \mu C_i \qquad (3.8)$$

where $0 < \alpha_i < 1$ and $i = 1, \ldots, M$. This has the effect of limiting the average utilization $\rho_i = \lambda_i/\mu C_i$ to a maximum of α_i for each link, a feature which is desirable in practice. In terms of the $r_{\ell}(j,k)$ and n_{jk}^{ℓ}, the set of constraints defined by Eq. (3.8) can be expressed as

$$\sum_{i,j,k,\ell} r_{\ell}^{i}(j,k)\gamma_{jk} \leq \alpha_i \mu C_i \qquad (3.9)$$

where $i=1, \ldots, M$. For each i, the summation in Eq. (3.9) is over all j,k,ℓ which involve flow on link i. Note that the requirement $\lambda_i \geq 0$ is automatically satisfied in Eq. (3.9) since $r_{\ell}^{i}(j,k) \geq 0$ for all i,j,k,ℓ. The problem of minimum hop flow assignment is now seen to be one of determining the set of routing variables $r_{\ell}(j,k)$ to minimize J in Eq. (3.5) subject to the nonnegativity and equality constraints in Eq. (3.6) and the inequality constraints in Eq. (3.9). This is a well-posed linear programming problem [2-15] which can be solved directly. In this work, however, a dynamic programming approach [16,17] which offers certain computational advantages over other techniques for this particular problem is developed.

The development proceeds by first forming the augmented performance function.

$$L = J - \sum_{\ell=1}^{m} \beta_\ell \left[\sum_{j,k,\ell} r_\ell^i(j,k)\gamma_{jk} - \alpha_i \mu C_i \right] \tag{3.10}$$

subject to the M-m constraints

$$\sum_{i,\ j,k,\ell} r_\ell^i(j,k)\gamma_{jk} \leq \alpha_i \mu C_i \tag{3.11}$$

In Eq. (3.10), J is given by Eq. (3.7) and the β_ℓ are Lagrange multipliers, while in Eq. (3.11), $i = m + 1, \ldots, M$. In this formulation, the M link capacity constraints in Eq. (3.11) are separated into two sets of size m and M-m, respectively, where $0 \leq m \leq M$. The value of m partitions the solution space into a routing variable space and a Lagrange multiplier space. Computationally, this serves to trade computer memory against repetitive calculations. Hence, the choice of m may be governed by available computing resources. When the latter is not an issue, one may choose m, $0 \leq m \leq M$, as a matter of convenience.

Referring to Eq. (3.6b), one sees that the dimensionality of the optimization problem can be reduced by an amount equal to the number of (j,k) source-destination pairs. Namely, one can set

$$r_\alpha(j,k) = 1 - \sum_{p \neq \alpha} r_p(j,k) \tag{3.12}$$

for some α for each (j,k) pair. Since the choice of α is arbitrary, without loss of generality, take $\alpha = 1$ for all (j,k). Substitution of Eq. (3.12) into Eq. (3.7) then gives

$$J = \frac{1}{\gamma} \sum_{j,k} \gamma_{jk} n_{jk}^{1} - \frac{1}{\gamma} \sum_{j,k} \gamma_{jk} \left[\sum_{p \neq 1} r_p(j,k) \quad n_{jk}^{1} \right]$$

$$+ \frac{1}{\gamma} \sum_{\substack{\ell,j,k \\ \ell \neq 1}} \gamma_{jk} r_{\ell}(j,k) n_{jk}^{\ell}$$

$$= \sum_{j,k} \frac{\gamma_{jk}}{\gamma} n_{jk}^{1} - \sum_{\substack{\ell,j,k \\ \ell \neq 1}} \frac{\gamma_{jk}}{\gamma} (n_{jk}^{\ell} - n_{jk}^{1}) r_{\ell}(j,k) \tag{3.13}$$

By using x_q, $q = 1, \ldots, W$, to denote the routing variables $r_{\ell}(j,k)$ over all ℓ,j,k with $i \neq 1$, and letting a_q denote the corresponding coefficients

$$\frac{\gamma_{jk}}{\gamma} (n_{jk}^{\ell} - n_{jk}^{1})$$

again over all ℓ,j,k with $i \neq 1$, Eq. (3.13) can be written as

$$J = K + \sum_{q=1}^{W} a_q x_q \tag{3.14}$$

In Eq. (3.14), $x_q \geq 0$ and the value of W is the number of routing variables in the original problem formulation minus the number of (j,k) source-destination pairs. Also, K denotes the value of the first term on the right-hand side in Eq. (3.13). Since the latter is independent of the routing variables, it shall be ignored in the development below.

In a similar fashion, the capacity constraints in Eq. (3.9) can be expressed by the relation

$$\sum_{q=1}^{W} a_{iq} \leq c_i \qquad (3.15)$$

for i=1, ..., M. Here, each a_{iq} takes on the appropriate value from the set $\{0, \pm \gamma_{jk}\}$ and each c_i is of the form

$$\alpha_i \mu C_i - \sum_{j,k} \gamma_{jk}$$

in accordance with the substitution of Eq. (3.12) for α=1 into Eq. (3.5) and rearrangement of the result into the form of Eq. (3.15).

As a result of Eqs. (3.14) and (3.15), the reduced dimension optimization problem is that of determining the set of routing variables x_q, q=1, ..., W, and the set of Lagrange multipliers, β_ℓ, ℓ=1, ..., m, to minimize

$$L' = \sum_{q=1}^{W} a_q x_q - \sum_{\ell=1}^{m} \beta_\ell \left[\sum_{q=1}^{W} a_{\ell q} x_q \right] \qquad (3.16)$$

subject to $x_q \geq 0$ and

$$\sum_{q=1}^{W} a_{iq} x_q \leq c_i \qquad (3.17)$$

where i = m+1, ..., M. It is a direct matter to show that this problem satisfies the conditions set forth in [18] for the existence and uniqueness of a fixed point solution for a global minimum in J.

From the dynamic programming view, this problem is one of determining a sequence of functions of M-m variables together with a search over an m-dimensional Lagrange multiplier space. The desired recurrence relation follows via application of the principle of optimality. While the development for arbitrary M and m is straightforward, it is lengthy and tedious, and will not be given here. Instead, a simple example is presented to indicate the dynamic programming approach, and then the general recurrence relation is cited.

Consider the case where M=4 and m=2 with W routing variables. Using Eq. (3.16) the recurrence relation is defined to be

$$f_s(c_3,c_4) = \min_{x_1,\ldots,x_s} \left[\sum_{q=1}^{s} a_q x_q - \beta_1 \sum_{q=1}^{s} a_{1q} x_q - \beta_2 \sum_{q=1}^{s} a_{2q} x_q \right] \quad (3.18)$$

for any s, $1 \leq s \leq W$, where the indicated minimization is subject to $x_q \geq 0$,

$$\sum_{q=1}^{s} a_{3q} x_q \leq c_3 \quad (3.19)$$

and

$$\sum_{q=1}^{s} a_{4q} x_q \leq c_4 \quad (3.20)$$

Rewriting the relations in Equations (3.18), (3.19) and (3.20) as

$$f_s(c_3,c_4) = \min_{x_s;x_1,\ldots,x_{s-1}} \Bigg\{ a_s q_s - \beta_1 a_{1s} x_s - \beta_2 a_{2s} x_s + \left[\sum_{q=1}^{s-1} a_q x_q - \beta_1 \sum_{q=1}^{s-1} a_{1q} x_c - \beta_2 \sum_{q=1}^{s-1} a_{2q} x_q \right] \Bigg\}$$

$$\sum_{j=1}^{s-1} a_{3q}x_q \leq c_3 - a_{3s}x_s$$

and

$$\sum_{j=1}^{2-1} a_{4q}x_q \leq c_4 - a_{4s}x_s$$

respectively, and applying the principle of optimality, yields

$$f_s(c_3,c_4) = \min_{x_s} \Big[a_s x_s - \sum_{\ell=1}^{2} \beta_\ell a_{\ell s} x_s + f_{s-1}(c_3 - a_{3s}x_s, c_4 - a_{4s}x_s) \Big] \qquad (3.21)$$

which is subject to $0 \leq x_s \leq \min\left(\frac{c_3}{a_{3s}}, \frac{c_4}{a_{4s}}\right)$.

Equation (3.21) is the desired recurrence relation for s-1, ..., W. Note that the solutions for f_s and x_s depend upon β_1 and β_2 as well as c_3 and c_4. The solution is concluded by varying the Lagrange multipliers β_1 and β_2 to satisfy the constraints

$$\sum_{q=1}^{W} a_{iq}x_q \leq c_i$$

for i=1, ..., 4.

For the general case, the dynamic programming recurrence relation is

$$f_s(c) = \min_{x_s} \Big[a_s x_s - \sum_{\ell=1}^{m} \beta_\ell a_{\ell s} x_s + f_{s-1}(c^*) \Big] \qquad (3.22)$$

for s=1, ..., W where the minimization over x_s is constrained by

$$0 \leq x_s \leq \min_i \left(\frac{c_i}{a_{is}} \right)$$

for i=m+1, ..., M. Also in Eq. (3.22), c and c* are (M-m)-dimensional vectors whose components are c_i and $c_i - a_{is}x_s$, respectively, for i=m+1, ..., M.

The solution of Eq. (3.22) yields the set of routing variables x_s as a function of c and the vector $\beta = (\beta_1, \ldots, \beta_m)$ of Lagrange multipliers, i.e., $x_s = x_s(c,\beta)$ for s=1, ..., W. The solution is completed by varying the Lagrange multipliers until the set of constraints

$$\sum_{q=1}^{W} a_{iq}x_q \leq c_i$$

where i = 1, ..., M, are satisfied (see Eq. (3.15)). This can be done using any appropriate linear search technique. Fibonacci search [16] has been applied to a number of examples with excellent results. Along with Eq. (3.12) for $\alpha=1$, this yields the entire set of routing variables $\{r_\ell(j,k)\}$ for minimum hop flow assignment.

The above procedure was applied to the network shown in Figure 3.2. The links are numbered 1 through 9 and the six nodes are labeled a through f. All link capacities μC_i, i=1, ..., 9, are taken to be 5 messages/sec with $\alpha_i = 0.8$ for all i, and four commodities are specified by the traffic requirements $\gamma_{ac} = \gamma_{ae} = 3$ and $\gamma_{be} = \gamma_{db} = 2$, each in messages/sec. The possible paths and their lengths are $n_{ac}^1 = 2$ via nodes abc, $n_{ac}^2 = 3$ via afdc, $n_{ae}^1 = 2$ via afe, $n_{ae}^2 = 3$ via afde, $n_{ae}^2 = 3$ via abce, $n_{ae}^3 = 4$ via afdce,

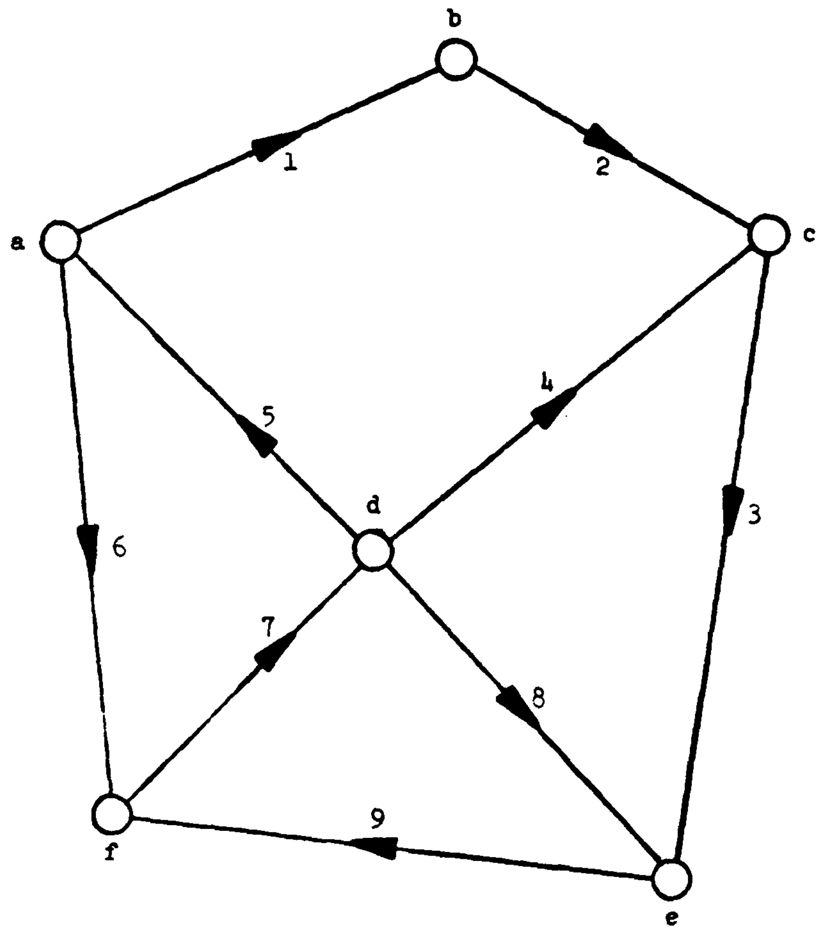

Figure 3.2 Network topology for DP Min-Hop example

$n^1_{be} = 2$ via bce, and $n^1_{db} = 2$ via dab. It should be noted that two of the ae paths have the same length.

The total number of routing variables is eight: $r_1(a,c)$, $r_2(a,c)$, $r_1(a,e)$, $r_2^6(a,e)$, $r_2^1(a,e)$, $r_3(a,e)$, $r_1(b,e)$, and $r_1(d,b)$. However, $r_1(b,e) = r_1(d,b) = 1$ since only one path each is allowed for (b,e) and (d,b) traffic, respectively. Further, from Eq. (3.12)

$$r_1(a,c) = 1 - r_2(a,c) \tag{3.23}$$

and

$$r_1(a,e) = 1 - r_2^6(a,e) - r_2$$
$$- r_3(a,e)$$

Hence, the reduced set of routing variables consists of the four elements $r_2(a,c)$, $r_2^6(a,e)$, $r_2^1(a,e)$, and $r_3(a,e)$. In this case, Eq. (3.13) is

$$J = 2 + \frac{3}{10}\Big[r_2(a,c) + r_2^6(a,e)$$
$$+ r_2^1(a,e) + 2r_3(a,e)\Big]$$

Since the network has nine links, there are $M = 9$ capacity constraints in the form of Eq. (3.15). For example,

$$-r_2(a,c) + r_2^1(a,e) \leq \frac{1}{3}$$

for link 1 and

$$r_2(a,c) \leq \frac{1}{3}$$

for link 6.

Application of the dynamic problem formulation with M=9 and m=2 provided the reduced set of four routing variables. Coupled with the single path routing variables for (b,e) and (d,b) traffic and the pair of constraints in Eq. (3.23), the complete set of routing variables was found to be

$$r_1(a,c) = 0.667 \qquad r_1(a,e) = 1.0$$
$$r_2(a,c) = 0.333 \qquad r_2^6(a,e) = 0$$
$$r_1(b,e) = 1 \qquad r_2^1(a,e) = 0$$
$$r_1(d,b) = 1 \qquad r_3(a,e) = 0$$

This minimum hop flow assignment yields J=2.1 as the minimum average path length over all source-destination pairs for this network. The choice of m=2 in this example made it possible to readily implement the algorithm on an HP 41C calculator.

3.2.3 Centralized Min-Hop with Delay Constraints

In the previous section a centralized Min-Hop routing assignment was developed without consideration for average message delay or average end-to-end message delay constraints. In the usual approach to flow assignment and routing in computer-communication networks the average message delay is minimized [5-8,19,20]. In this section, the approach will be to satisfy end-to-end message delay constraints which serve to meet user requirements for the timely delivery of messages. The latter is of particular importance where critical source-destination node pairs are involved. The algorithm to be presented consists of two parts, the first part having been developed in the previous section. After presentation of the algorithm, the results are illustrated via a network example.

The problem formulation is that of the preceding section. The function to be minimized is Eq. (3.5) subject to the link capacity constraints of Eq. (3.8). In addition, a set of constraints each of the form

$$\sum_{\substack{i \in \pi_{ab}^{\ell} \\ \ell \in \pi_{ab}}} \frac{r_{\ell(a,b)}}{\mu C_i - \lambda_i} \leq T_{ab} \qquad (3.24)$$

is imposed upon the solution set. For each source-destination pair (a,b) of interest, Eq. (3.24) defines an end-to-end average message delay constraint. Eq. (3.24) and the associated message routing variables are discussed in Section 1.4.3. As noted earlier, the solution is also subject to conservation of total average flow in the λ_i and also in the individual commodities. The algorithm is now described in two parts. Part A establishes the initial feasible flow and Part B optimizes the initial flow to satisfy the delay constraints of Eq. (3.24). The algorithm is as follows:
follows:

A. Initial feasible flow

1. Use the method of Section 3.2.2 to establish the initial flow

$$f^{(0)} = [\lambda_1^{(0)} \lambda_2^{(0)} \ldots \lambda_M^{(0)}]$$

and the set of variables $\{r_\ell^{(0)}(j,k)\}$ for the entire network as well as for each end-to-end delay constraint (a,b). ($r_\ell^{(0)}(j,k)$ denotes the zeroth iteration of $r_\ell(j,k)$).

2. If this initial flow satisfies the end-to-end delay constraints in Eq. (3.24), stop. Otherwise proceed to Part B.

B. Flow Optimization

1. Given $f^{(n)}$ and $\{r_\ell^{(n)}(a,b)\}$ from the nth iteration, n = 0, 1,..., evaluate the left-hand side in Eq. (3.24) for each (a,b) pair and denote the result by T'_{ab}.

2. For each T'_{ab} which violates its constraint, choose the second shortest path (as obtained in Part A above) and deviate $h\gamma_{ab}$, $0 < h < 1$, onto that path to form $T'_{ab}(h)$. Choose $h \epsilon (0,1)$ so that T'_{ab} satisfies its constraint. If necessary, deviate flow onto the third shortest (a,b) path and so on until the constraint is satisfied, or the set of successive shortest (a,b) paths is exhausted and the constraint cannot be satisfied. In the latter case, terminate the process. Otherwise, proceed to Step 3.

3. The λ_i and $r_\ell(a,b)$ so determined, together with those unaffected by calculations in Step 2, if any, define $f^{(n+1)}$ and $\{r_\ell^{(n+1)}(j,k)\}$. If these results

satisfy all end-to-end delay constraints, stop.

Otherwise, return to Step 1 and repeat the process.

The algorithm was applied to the network shown in Figure 3.3 where the underlined values are the link capacities μC_i in messages/sec. The assumed traffic requirements were $\gamma_{ab} = \gamma_{cd} = 10$, $\gamma_{bh} = \gamma_{eb} = \gamma_{fb} = \gamma_{gf} = 5$, and $\gamma_{ca} = 1$, each in messages/sec. The end-to-end average delay constraint values were $T_{ab} = T_{cd} = 1.0$ sec, and α_i was chosen to be 0.8 for each link, except for link cf was set to 0.9.

Part A of the algorithm led to the flow assignment given in the figure by the first number in parentheses for each link. The corresponding routing was

γ_{ab}: 8 via aedb
2 via afdhb

γ_{cd}: 10 via cfd

γ_{bh} 5 via bdh

γ_{eb}: 5 via edhb

γ_{fb}: 5 via fcgb

γ_{gf}: 5 via ghf

γ_{ca}: 1 via cfa

in messages/sec, the average path length was $\bar{n} = J = 2.537$, and the two end-to-end average delays were $T'_{ab} = 1.14$ and $T_{cd} = 1.33$ sec.

Since both end-to-end average delay constraints were violated, Part B of the algorithm was employed to satisfy the constraints. The resulting flow assignment is given in the figure by the second number in parentheses for each link, and the associated routing is

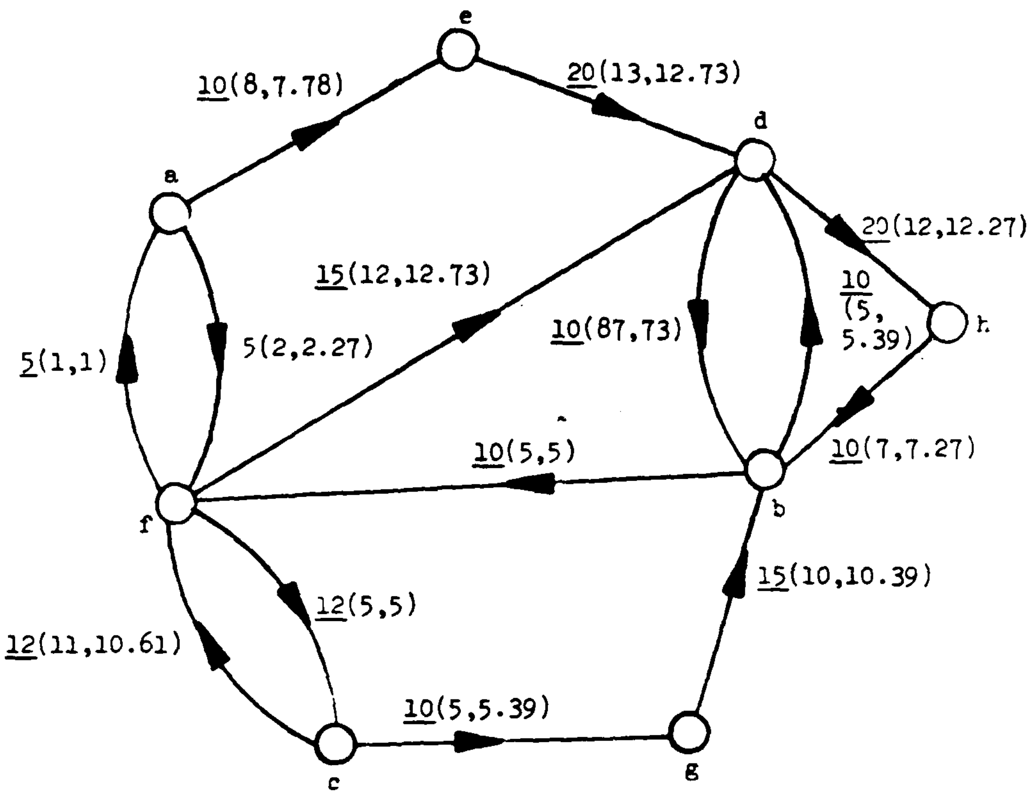

Figure 3.3 Network example for centralized Min-Hop algorithm constrained by average end-to-end message delay

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MODELING AND PERFORMANCE OPTIMIZATION OF LARGE-SCALE DATA-COMMU--ETC(U)
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γ_{ab}: 7.73 via aedb
2.27 via afdhb

γ_{cd}: 9.61 via cfd
0.39 via cgbd

γ_{bh}: 5 via bdh

γ_{eb}: 5 via edhb

γ_{fb}: 5 via fcgb

γ_{gf}: 5 via gbf

γ_{ca}: 1 via cfa

The average path length increased slightly to $\bar{n} = J = 2.573$ and the two constraints were satisfied to two decimal places, namely, $T'_{ab} = T'_{cd} = 1.00$ sec.

The algorithm converged in one pass through Part A and two iterations in Part B, the stopping condition for the latter being satisfaction of the end-to-end delay constraints to two decimal places. No attempt was made to optimize the algorithm in any sense. The calculations required in Step 3 of Part A and in Step 2 of Part B were done via a direct search.

3.2.4 Decentralized Min-Hop Strategy

In the preceding sections a theory and procedure for constrained Minimum Hop routing has been developed for use in large-scale, data-communication networks. The concern of this section is with achieving the same result employing a distributed algorithm which is implemented nodewise over the entire network with each node requiring information only from its adjacent neighboring nodes. The approach taken for this problem is fundamentally different from that used for the centralized problem in [1,2], there being no readily apparent extension or modification of the latter to the distributed routing problem.

The organization of this section is as follows. First, the constrained Min-Hop routing problem is formulated from the perspective of distributed computations with "nearest-neighbor" (or "adjacent") information. A two-part algorithm is then developed in which the first part provides unconstrained, alternate path, Minimum Hop routing and the second part adjusts this routing to satisfy the end-to-end average message delay constraints. The section concludes with an example to illustrate the algorithm.

The methodology which was developed in [8] for distributed minimum average delay routing in acyclic networks is followed in part. Consider a message-switched network with N nodes and M directed links in which message lengths are independent and exponentially distributed, and network message arrival processes are independent and Poisson. Denote by γ_{ik} the average rate in messages/sec at which traffic enters the network at node i destined for node k, and take $\gamma_{jk} = 0$ for all i = k.

In general, i, j, and k denote nodes, and whether they refer to source, destination or intermediate nodes will be clear from context as was the case above. With this in mind, let C_{ij} be the capacity in messages/sec, of the directed link (i,j) from any node i to any node j, i ≠ j. Similarly, use λ_{ij}, i ≠ j, to specify the average total flow on link (i,j) in messages/sec. Let $u_{ij}(k)$, i ≠ j and i ≠ k, denote the component of λ_{ij} whose destination is node k. Finally, let $r_n(i,k)$ be the routing variable for (i,k) source-destination traffic on the nth shortest (i,k) path. In connection with the latter, it is assumed that there is at least one directed, loop-free path from node i to node k, where by the term loop-free it is meant that no node is encountered more than once in traversing the path. In fact, all allowable source-destination paths are loop-free in this sense, and the network is accordingly acyclic in its flow.

The global problem is to minimize the average path length [10]

$$J = \frac{1}{\gamma} \sum_{i,j} \lambda_{ij} \qquad (3.25)$$

with respect to the set of routing variables $\{r_n(i,k)\}$. In Eq. (3.25), the summation is over all M links in the network and

$$\gamma = \sum_{i,k} \gamma_{ik}$$

where the latter summation is over all (i,k) source-destination node pairs. The minimization with respect to the $r_\ell(i,k)$ is over all (i,k) source-destination node pairs where n indexes the paths from shortest to longest in terms of the number of links in each for each i,k.

The above minimization is subject to a set of one or more end-to-end average message delay constraints

$$\sum_{(i,j)\epsilon\pi_{ab}} \frac{r_\ell(a,b)}{C_{ij} - \lambda_{ij}} \leq T_{ab} \qquad (3.26)$$

For each (a,b) source-destination node pair of interest in Eq. (3.26), T_{ab} is fixed, π_{ab} is the set of all links (i,j) which are involved in carrying traffic from node a to node b, π_{ab}^{ℓ} the set involved in the ℓth shortest path, and the $r_{\ell}(a,b)$ depend upon the routing assignment.

In addition to the constraint set specified by Eq. (3.26), the minimization of J in Eq. (3.25) is subject to conservation of total average flow in the λ_{ij} and also in the individual commodities. With respect to the former, the requirement is that $0 \leq \lambda_{ij} < C_{ij}$ for all (i,j) links. This is handled in the algorithm via the strict inequality constraint $0 \leq \lambda_{ij} \leq \eta_{ij}C_{ij}$ where $0 \leq \eta_{ij} < 1$, which has the effect of limiting the average utilization $\rho_{ij} = \lambda_{ij}/C_{ij}$ to no more than η_{ij} for each link.

The remaining constraints are those imposed by nonnegativity and conservation of flow on the routing variables. These lead, respectively, to the relations

$$r_{\ell}(i,k) \geq 0 \tag{3.27a}$$

and

$$\sum_{\ell} r_{\ell}(i,k) = 1 \tag{3.27b}$$

for all ℓ, i, k in Eq. (3.27a) and for each (i,k) pair in Eq. (3.27b).

The development of this approach begins by noting that since the network is acyclic in its flow, it can be depicted as shown in Figure 3.4 using virtual node splitting where necessary. In the figure, the nodes are grouped as shown and indexed from 1 to S, right-to-left. All nodes in Group 1 are destination nodes, while those in Groups 2 through S-1 may be source and/or destination and/or switching nodes. Nodes in Group S are all source nodes.

As a result of the structure in Figure 3.4, a natural nodewise decomposition of the network is that given in Figure 3.5 for a representative node i where the adjacent upstream nodes are indexed by m, the adjacent downstream nodes by j, and the downstream destination nodes by k. For purposes of the algorithm given below, it is necessary to assume that the adjacent downstream nodes {j} can communicate with node i for purposes of routing information exchange.

<u>Step 1</u> of the algorithm is a distributed procedure for determining all source-destination paths and their respective lengths. To determine the paths and their lengths, let $\{\pi_{ik}: i \neq k$, k a destination node, and i any node in Groups 2 through S in Figure 3.4} be the set of all (i,k) paths in the network. Then define path length by

$$\alpha_{ij}(k) = \sum_{\ell_n \epsilon \pi_{ik}} \ell_n \tag{3.28}$$

where $i \neq j$ and j ranges over the set of adjacent nodes downstream from node i as indicated in Figure 3.5. In Eq. (3.28), i ranges over all nodes in each group, but calculations for each i in a group are independent of those at all other nodes in the group. Also, $\alpha_{ij}(j) = 1$, $i \neq j$, and n ranges over π_{ik} with $\ell_n = 1$ for each link.

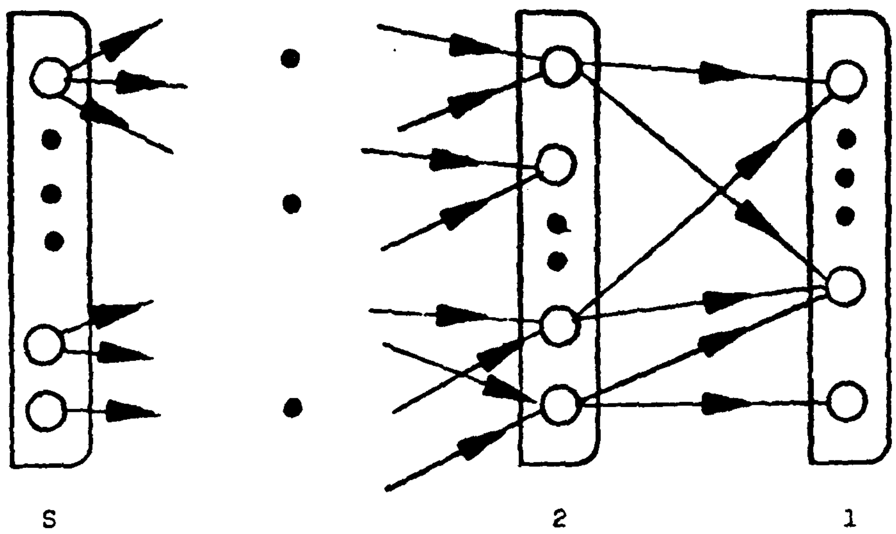

Figure 3.4 Node grouping is shown for an acyclic network

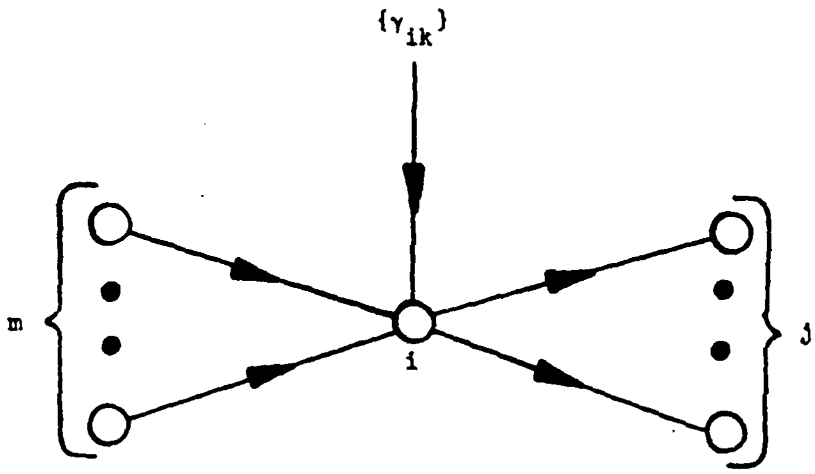

Figure 3.5 A representative node for an acyclic network is shown

The process of determining the $\alpha_{ij}(k)$ begins in Group 2 and sweeps, synchronously or asynchronously, from right to left, terminating at Group S (see Figure 3.4). At this point, each source node in the network knows its set of (i,k) source-destination paths and their respective lengths.

Step 2 of the algorithm assigns the routing variables $\{r_\ell(i,k)\}$ for the entire network. For the minimum $\alpha_{ij}(k)$, i a source node, $r_1(i,k)$ is assigned to the link (i,j). For the next largest $\alpha_{ij}(k)$, $r_2(i,k)$ is assigned in the same way, and the procedure continues at each i until the set of $\alpha_{ij}(k)$ is exhausted. This process begins at the nodes in Group S in Figure 3.4 and moves stepwise left to right until it reaches Group 2 at which point all possible $r_\ell(i,k)$ are now indexed to the appropriate outbound links of all source and intermediate nodes.

Step 3 involves the assignment of the $u_{ij}(k)$ over the entire network to minimize the average path length subject to the capacity constraints and the conservation of flow. Owing to the nodewise decomposition in Figure 3.5, the global optimization problem specified by Eq. (3.25) simplifies to the following linear programming subproblem at each node i:

$$\min_{\{u_{ij}(k)\geq 0\}} \sum_{j,k} \alpha_{ij}(k) u_{ij}(k) \qquad (3.29)$$

subject to

$$\sum_k u_{ij}(k) \leq n_{ij} C_{ij} \qquad (3.30)$$

and

$$\sum_{j} u_{ij}(k) = \sum_{\substack{m \\ k \neq 1}} u_{mi}(k) + \gamma_{jk} \tag{3.31}$$

Calaulations commence at the nodes in Group S in Figure 3.4 and move to the right, terminating at the nodes in Group 2. The information exchange required consists of the $u_{mi}(k)$ being provided to node i from the calculations carried out at the adjacent upstream nodes {m} (see Figures 3.4 and 3.5).

Determination of the routing variables now follows immediately. Starting at a source node i, the routing variable $r_\ell(i,k)$ for the ℓth shortest (i,k) path is

$$r_\ell(i,k) = \frac{u_{ij}(k)}{\gamma_{ik}} \tag{3.32}$$

Again, the procedure sweeps from left to right, terminating at Group 2.

This completes the first part of the algorithm wherein the minimum hop routing has been achieved subject to flow conservation and link capacity constraints. The next and final task is to adjust this routing to satisfy the set of end-to-end average delay constraints which are specified by Eq. (3.25).

<u>Step 4</u> of the algorithm requires that the source nodes {a} calculate their respective $\{T'_{ab}\}$ which result from the routing assignment in Step 3 above, and compare them with the specified $\{T_{ab}\}$. This is done termwise for all a,b source-destination pairs via a sweep from right to left beginning at Group 2 in Figure 3.4

and terminating at Group S using the left-hand side in Eq. (3.25). If the result is that $T'_{ab} \leq T_{ab}$ for all (a,b), the process terminates. If not, proceed to the next step.

Step 5 pertains to all nodes i which are involved in the routing of (a,b) traffic wherein $T'_{ab} > T_{ab}$. Referring to Figure 3.5, solve the following nonlinear programming subproblem:

$$\min_{\{u_{ij}(k) \geq 0\}} \sum_{j,k} \zeta_{ij}(k) u_{ij}(k) \tag{3.33}$$

subject to

$$\sum_{k} u_{ij}(k) \leq n_{ij}\ C_{ij} \tag{3.34}$$

and

$$\sum_{j} u_{ij}(k) = \sum_{\substack{m \\ k \neq 1}} u_{mi}(k) + \gamma_{ik} \tag{3.35}$$

The analogy to the subproblem specified by Eqs. (3.29), (3.30) and (3.31) is clear. However, the path length metric here is that of average link delay. In particular, the $\zeta_{ij}(k)$ in Eq. (3.33) are defined by

$$\zeta_{ij}(k) = \sum_{\ell_n \epsilon \pi_{ik}} \ell_n \tag{3.36}$$

where

$$\ell_n = \frac{r_n(i,k)}{C_{ij} - \lambda_{ij}} \tag{3.37}$$

and π_{ik} is the same as in Eq. (3.28). In Eq. (3.37), the values of $r_n(i,k)$ and λ_{ij} which are used to evaluate ℓ_n are those obtained in the first part of the algorithm.

Determination of the $\zeta_{ij}(k)$ proceeds in the same fashion as that given for the $\alpha_{ij}(k)$ in Step 1 except that the link length is now given by Eq. (3.37).

Solution of the subproblems specified by Eqs. (3.33), (3.34), and (3.35) commences at Group S and moves to Group 2. The process, however, is carried out only at those nodes which participate in end-to-end delay constraint violations.

Finally, the new end-to-end delay constraints are calculated and compared with the specified ones. At this point, the algorithm has either satisfied all end-to-end delay constraints or found that one or more of them cannot be met. This concludes the algorithm. However, it should be noted that the solution procedure for the second part of the algorithm is heuristic. It should be clear that this second part is suboptimal in the sense that J in Eq. (3.25) is not minimized over the entire network in satisfying the delay constraints. On the other hand, its virtue resides in its simple implementation which is attractive for applications.

The above algorithm was used to solve the constrained minimum hop routing problem for the network shown in Figure 3.6 where the underlined values are the link capacities in messages/sec. The assumed traffic requirements were $\gamma_{ab} = \gamma_{cd} = 10$, $\gamma_{bh} = \gamma_{eb} = \gamma_{fb} = \gamma_{gf} = 5$, and $\gamma_{ca} = 1$, each in messages/sec. There are thus 7 commodities. The end-to-end average message delay constraints were taken to be $T_{ab} = T_{cd} = 1.0$ sec, and we set $\eta_{ij} = 0.8$ for each link with the exception of $\eta_{cf} = 0.9$.

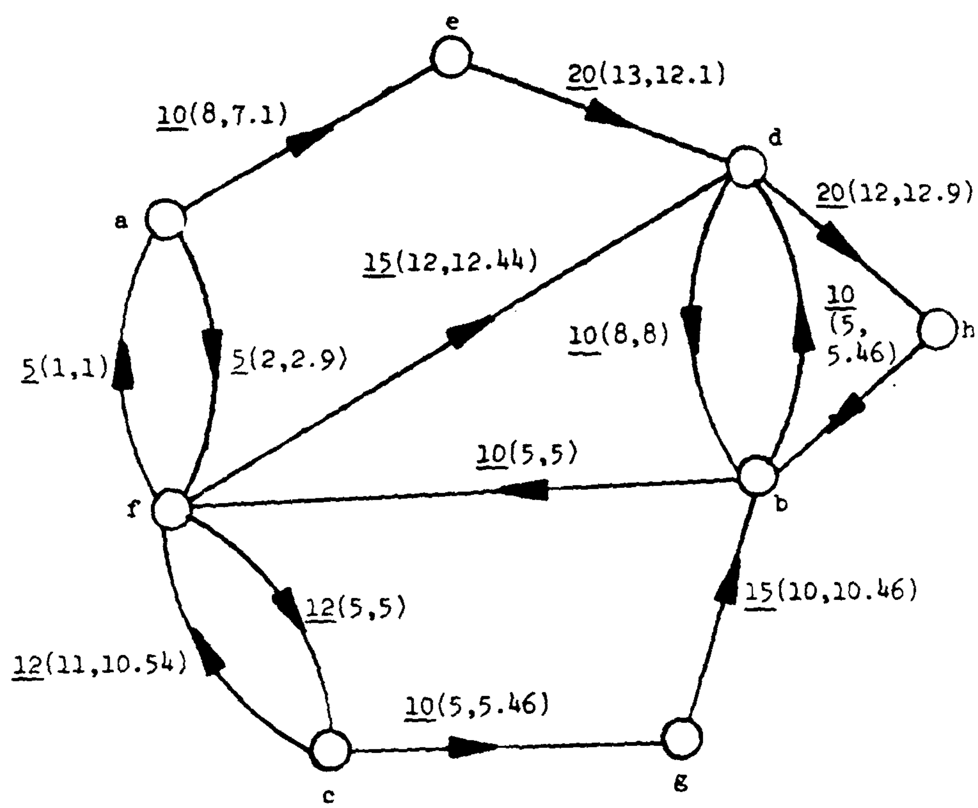

Figure 3.6 The network example employed to illustrate the decentralized Min-Hop algorithm

The minimum hop flow assignment found using the first part of the algorithm is given by the first number in the parenthesis associated with each link, the units being messages/sec. The corresponding routing was

γ_{ab}: 8 via aedb 2 via afdhb	γ_{eb}: 5 via edhb
	γ_{fb}: 5 via fcgb
γ_{cd}: 10 via cfd	γ_{gf}: 5 via ghf
γ_{bh}: 5 via bdh	γ_{ca}: 1 via cfa

in messages/sec. The average path length was J = 2.537 links, and the two end-to-end average delays were T'_{ab} = 1.14 sec and T'_{cd} = 1.33 sec.

The second part of the algorithm was then applied successively to the (a,b) and (c,d) flows to meet the delay constraints to two decimal places, viz., $T_{ab} = T_{cd} = 1.00$ sec. The resulting flow assignment is given in the figure by the second number in parenthesis for each link, the associated routing being

γ_{ab}: 7.1 via aedb 2.9 via afdhb	γ_{eb}: 5 via edhb
	γ_{fb}: 5 via fegb
γ_{cd}: 9.54 via cfd 0.46 via cgbd	γ_{gf}: 5 via gbh
γ_{bh}: 5 via bdh	γ_{ca}: 1 via cfa

In satisfying the delay constraints, the average path length increased to J = 2.593 links.

Solution of this same network routing problem via the centralized algorithm in [1] gave exactly the same results for the unconstrained minimum hop part. However, the centralized algorithm, which is known to be optimal for both parts, led to a slightly different routing in satisfying the constraints and gave J = 2.573 links. While the difference is not significant for this example, it could be for others and points to the need, noted earlier, for an exact distributed algorithm to satisfy the delay constraints.

3.3 Path Rate Distortion Equalization (PRDE) Strategy

The PRDE strategy is to set the effective link capacity $C_i/R^i(D^i)$ equal to a constant C for all links in the network. The objective of this strategy, then, is to balance the link capacities throughout the network. This is similar in concept to the load-leveling strategies employed in multiple processor environments, telecommunication systems, and multiple disk string management.

The primary consequence of the strategy is to allow for a simple capacity assignment methodology as will be shown in the following paragraphs. The development of this section is as follows. First, a brief overview of an implementation scheme for this strategy is presented. The section then concludes with a mathematical development of capacity assignment under the PRDE strategy.

Perhaps the simplest method for network level implementation of the PRDE policy is the use of identical communication links throughout the network, e.g., conditioned 9600 baud land-lines. Identical communication links guarantee that, in the presence of identical switching processors and balanced message flows with identical source message probabilities, that the rate distortion functions $R^i(D^i)$ are identical. This assumption would, in practice, be valid for many electronic mail systems and local area networks. On the other hand, the assumption would be invalid for shared resource networks servicing diverse users, e.g., Tymnet. A particularly promising test bed for this strategy would be a local-area electronic mail network.

One of the more difficult network design problems is the optimum selection of link capacities. In a network employing PRDE, the capacity assignment is simplified. For this problem, the network model of Section 3.2.2 is invoked and the solution methodology is, in part, developed in [Chapter 5, 10]. The capacity assignment is solved for a fixed network topology and known traffic flow $\{\lambda_i\}$. The objective of the solution is to choose the link capacities $\{C_i\}$ so as to minimize the average message delay T subject to a linear capacity cost S. The form for T is given in Eq. (1.25). The linear capacity costs are

$$S = \sum_{i=1}^{M} d_i \left(\frac{C_i}{R^i(D^i)} \right) \tag{3.38}$$

where d_i is the cost for each unit of effective capacity in the ith link. The solution involves using Lagrange multipliers. Forming the augmented function

$$L = T + \beta \left[\sum_{i=1}^{M} d_i \frac{C_i}{R^i(D^i)} - S \right] \tag{3.39}$$

where ψ is the undetermined Lagrange multiplier and taking the derivative with respect to effective capacity $\{C_i/R^i(D^i)\}$, yields the following set of equations

$$\frac{\partial L}{\partial C_i} = 0 \tag{3.40}$$

for i = 1,2,...M.

Simplification of Eq. (3.40) gives

$$\frac{C_i}{R^i(D^i)} = \frac{\lambda_i}{\mu} + \frac{1}{\sqrt{\beta\gamma\mu}} \left(\frac{\lambda_i}{d_i}\right)^{1/2} \tag{3.41}$$

for i = 1,2,...M.

The Lagrange multiplier β in Eq. (3.41) is evaluated by multiplying Eq. (3.41) by d_i and summing on i. The optimal capacity assignment under the PRDE policy is then found to be

$$C_i = R^i(D^i) \left\{ \frac{\lambda_i}{\mu} + \frac{\left[S - \sum_{i=1}^{M} \frac{\lambda_i d_i}{\mu}\right]}{d_i} \frac{\sqrt{\lambda_i d_i}}{\sum_{j=1}^{M} \sqrt{\lambda_j d_j}} \right\} \tag{3.42}$$

As can be seen from Eq. (3.42) the optimal capacity assignment for minimizing average message delay is dependent on link flow, average message length, cost criteria, and the link rate distortion function. The implementation of PRDE can now take place by either choosing the desired message distortion and then calculating the link capacity using Eq. (3.42) or assuming a constant effective capacity C for all links and choosing both link capacity and message distortion. In the latter technique, if the constant C, the link capacity C_i, and the distortion D_i, are properly chosen such that Eq. (3.42) is valid for all links, then the PRDE policy will lead to the minimum average message delay.

3.4 Path Rate Prioritization (PRDF) Strategy

As noted in Chapter 2, the objective of this policy is to give selected commodities preference over other commodities by maximizing $C_{jk}/R_{jk}(D)$ for some (j,k) pairs. The consequences of this policy depend on the method of implementation. In the following paragraphs, two implementation schemes will be outlined and their associated consequences discussed. In the first implementation scheme, let $C_{jk}/R_{jk}(D)$ be maximized by allowing increased message distortion D. There are two effects: first, obviously, the (j,k) message traffic will be degraded, i.e., contain more errors, second, the channel queueing delay will be reduced. The former effect will not be acceptable if higher priority messages are also intended to be low distortion messages. The latter effect, which is quantified in Chapter 4, is, in general, desireable. Finally, this scheme would be relatively easy to implement in a large-scale network, although it may not be optimal from a data integrity viewpoint.

The second implementation policy makes use of the set of path capacities C_{jk}. By upgrading the links involved in the priority traffic, message delay will be decreased without sacrificing message integrity. This approach, in fact, is commonly used in systems such as AUTOVON where high-quality communications are desired with a minimum of message delay. The penalty for this approach, of course, is in the higher costs incurred in the operation of high capacity links. In general, such a policy could only be pursued to meet special user requirements.

3.5 Conclusion

In this chapter, three routing strategies which are relevent to the modeling of data-communications networks via information-theoretic methodologies have been presented. The focus of the chapter was on the Minimum-Hop routing strategy because:

1. The strategy optimized an important performance metric.
2. The strategy could be used with end-to-end message delay constraints.
3. The strategy could be implemented in either a centralized or distributed fashion.

In addition to the Min-Hop routing strategy, two other strategies were presented. In particular, the capacity assignment under PRDE was developed and presented, and the limitations of PRDP were presented.

From the research contained in this chapter certain intriguing questions become apparent and offer themselves as directions for future investigations. These questions will be addressed in Chapter 5.

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Chapter 4

ADDITIONAL COMPUTATIONAL RESULTS

4.1 Introduction to Chapter 4

In this chapter, the theory developed in Chapters 2 and 3 is applied to a link example and a network example to provide illustrations of key results. In Section 4.2, a generalized information-theoretic calculation of link capacity C_i is presented. Also in Section 4.2, the distortion measures introduced in Chapter 2 are used to calculate the link rate distortion functions $R^i(D^i)$. In Section 4.3, the average link queueing delay $T_i(\lambda_i, C_i, R^i(D^i))$ is calculated and displayed for a typical link as a function of normalized distortion D_N^i and message rate λ_i under various distortion measures. Section 4.4 illustrates, via a network example, key results in the implementation of path level rate distortion. The average end-to-end delay of a single commodity, an upper bound for the average end-to-end delay, and the average message delay for all commodities is calculated and presented as a function of message corruption. The chapter concludes with Section 4.5 and a best case-worst case comparison of delay as a function of distortion.

4.2 $R^i(D^i)$ for Eight Distortion Measures

In Chapter 2, four basic context-free distortion metrics and their associated normalized forms have been proposed as measures for the costs of message corruption. Briefly recapitulating, metrics 1 and 2 measure message errors from a bit-level perspective, with and without an attached message priority, respectively. Metrics 3 and 4 assign distortion costs from a character-level perspective, again with and without attached message priority, respectively. The normalized forms of the four basic distortion measures have been introduced to facilitate network design and provide a basis for performance comparisons. These eight distortion measures are used in the calculation of the link rate distortion function $R^i(D^i)$.

The link rate distortion function, defined in Eq. (2.35), is a convex linear programming problem subject to both equality and inequality constraints. An iterative algorithm due to Blahut and described in [1] may be used to solve the problem for general distortion measures and message input probabilities $\{P_s^i(j,k)\}$. Analytical techniques for the calculation of $R^i(D^i)$ are described in [1-3]. These techniques generally exploit symmetry properties in the message probability distributions or in the matrix of distortion metrics. For the examples of this section, the computational technique introduced in [2] and elaborated on in [3] was utilized.

To illustrate the differences in the proposed distortion measures, assume that in the network for this example it is desired to transmit 256 binary coded messages. Further, assume that in this network the 256 messages are all equally likely to be transmitted by the individual users, i.e.,

$$P_s^i(j,k) = \frac{1}{256} \qquad (4.1)$$

for all i, s, and (j,k) pairs. In addition, the network character code is comprised of 4 bit words, e.g., a 16-bit message is made up of 4 characters. Before the distortion measures can be quantified further, it is necessary to calculate the required message length in bits. Intuitively, one sees that under binary coding, 256 equally likely messages can be obtained using 8 bits, because

$$2^8 = 256$$

However, the same result can be obtained from an application of Eq. (2.49), as the following calculation quickly shows. Let each link in the example network be a binary-symmetric memoryless channel. Then for the symmetric discrete memoryless channel (DMC) with N output messages and N input messages and transition probabilities given by

$$Q_{j|i} = [1 - (N - 1)\varepsilon]\delta_{ij} + \varepsilon(1 - \delta_{ij}) \qquad (4.2)$$

where δ_{ij} is the Kronecker delta function, the capacity as defined by Eq. (2.49) becomes

$$\tilde{C}_i = \log N - H(\varepsilon) \text{ bits/channel usage} \qquad (4.3)$$

where for binary coding log N is to base 2 and $H(\cdot)$ is the usual entropy function. For a distortionless channel $\varepsilon = 0$ and $H(0) = 0$ so that Eq. (4.3) becomes

$$\tilde{C}_i = \log N = \log 256 = 8 \text{ bits/channel usage} \qquad (4.4)$$

(The above result for N messages is an extension of the binary channel developed in [4].)

Using the above result, the distortion measures will be of the form,

$$d(\vec{m},\vec{z}) = \frac{1}{8}\, d_m + P \qquad (4.5)$$

for the binary level, and

$$d(\vec{m},\vec{z}) = \frac{1}{2}\, d_m^c + P \qquad (4.6)$$

for the character level. For the binary level calculation, d_m will range from 0, for messages received without errors, to 8 for messages which have 8 errors. For the character level measure, the range is from 0 to 2 corresponding to the message with no errors to a message with 2 character errors. (Note that D_N^i denotes the normalized distortion. $R^i(D^i)$ will be used to denote the rate distortion function with distortion D^i or D_N^i. The correct distortion measure is clear from the context in which $R^i(D^i)$ is used.)

Under the above assumptions and conditions, the link rate distortion functions have been calculated and are shown in Figures 4.1 thru Figure 4.8. In Figures 4.1 and 4.2 the unnormalized and normalized binary level Hamming distance measure with no message priority was used. Note that $R^i(D^i)$ has the classical shape of the binary symmetric channel of capacity $\bar{C}_i = 1/2$. Figure 4.3 shows $R^i(D^i)$ for the unnormalized binary level measure with message priority P = 2. The normalized curve is shown in

Figure 4.4. The use of message priority has had the desired effect of increasing $R^i(D^i)$, over the range $0 < D_N^i < 1$, compared with the curve of Figure 4.2. Figure 4.5 and Figure 4.6 show the results for the character level metric with no message priority, unnormalized and normalized curves, respectively. Comparing the $R^i(D^i)$ for the character level measure with the $R^i(D^i)$ for the binary level measure, one sees that the character level measure leads to higher required message rates for equivalent distortion in the range [0,1]. This is a somewhat surprising result which may have a bearing on the issue of bit-oriented versus character-oriented protocols. Using the character level metric with message priority P = 2, the unnormalized and normalized $R^i(D^i)$ are shown in Figure 4.7 and Figure 4.8, respectively. Again, the use of message priority has had the desired effect of increasing $R^i(D^i)$ over the range $0 < D_N^i < i$.

In summary, the curves of $R^i(D^i)$ presented in this section provide evidence for the following conclusions:

1. The use of message priority in the distortion measure will act to decrease effective link capacity and increase message fidelity.

2. The character level measure does not offer effective link capacity improvements over the binary level measure. This result may have a bearing on the performance analysis of bit-oriented and character-oriented computer-communication protocols.

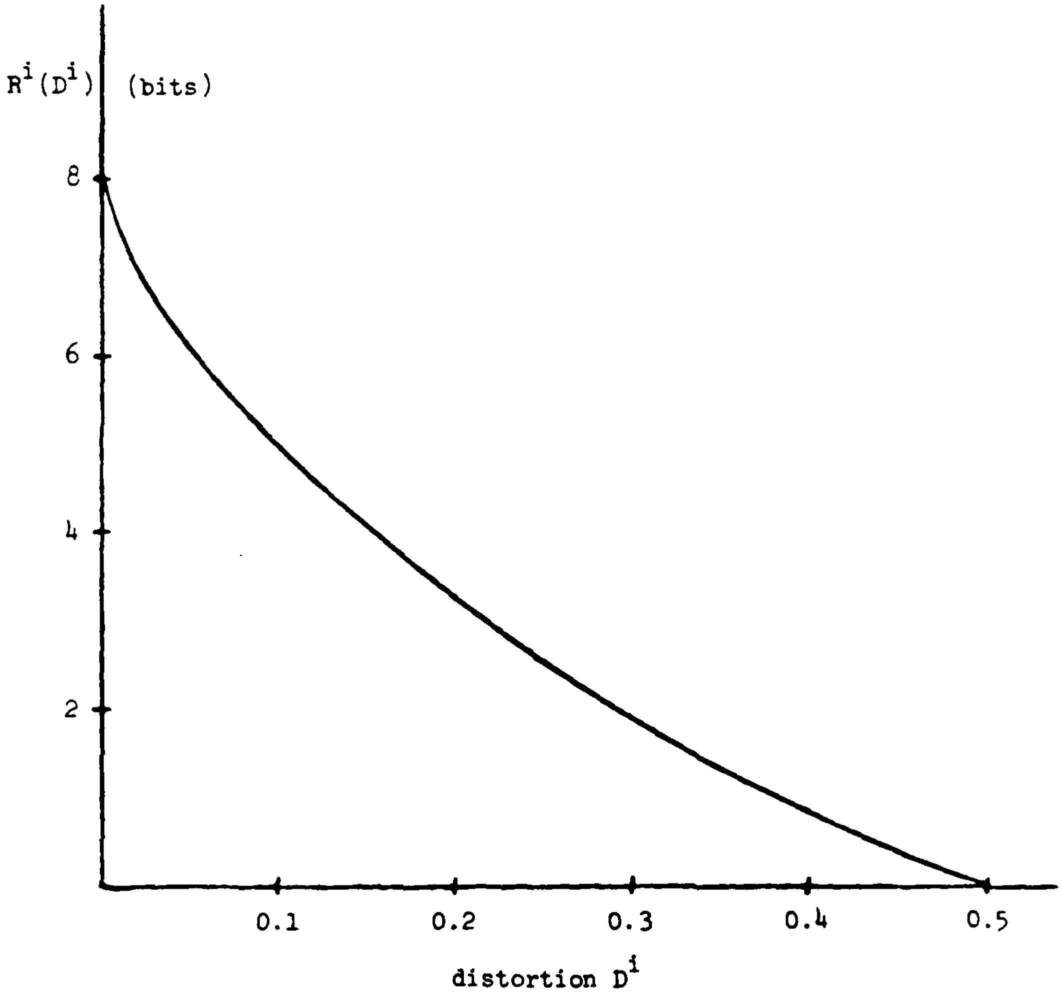

Figure 4.1 The unnormalized link rate distortion function $R^i(D^i)$ for the binary level distortion measure with message priority $P = 0$

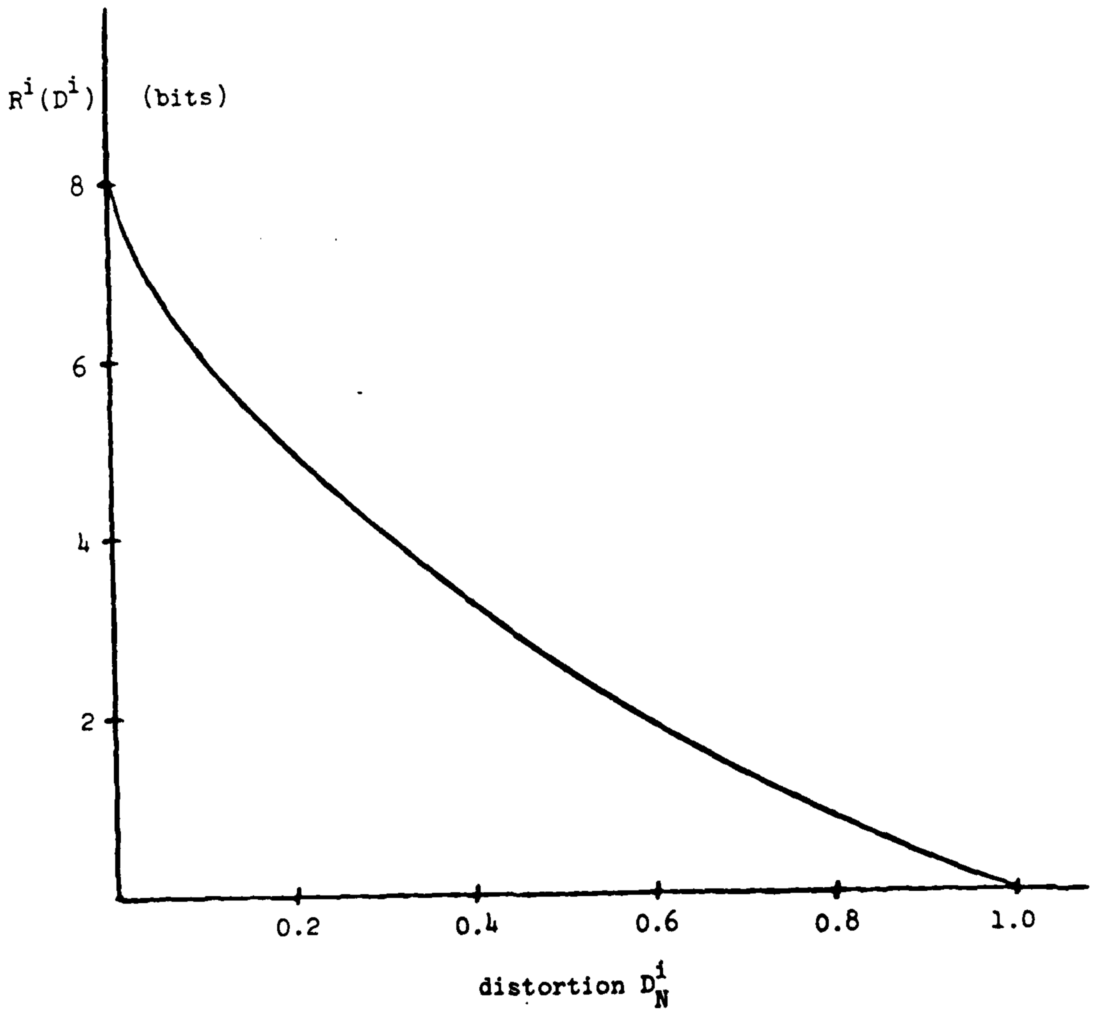

Figure 4.2 The normalized link rate distortion function $R^i(D^i)$ for the binary level distortion measure with message priority $P = 0$

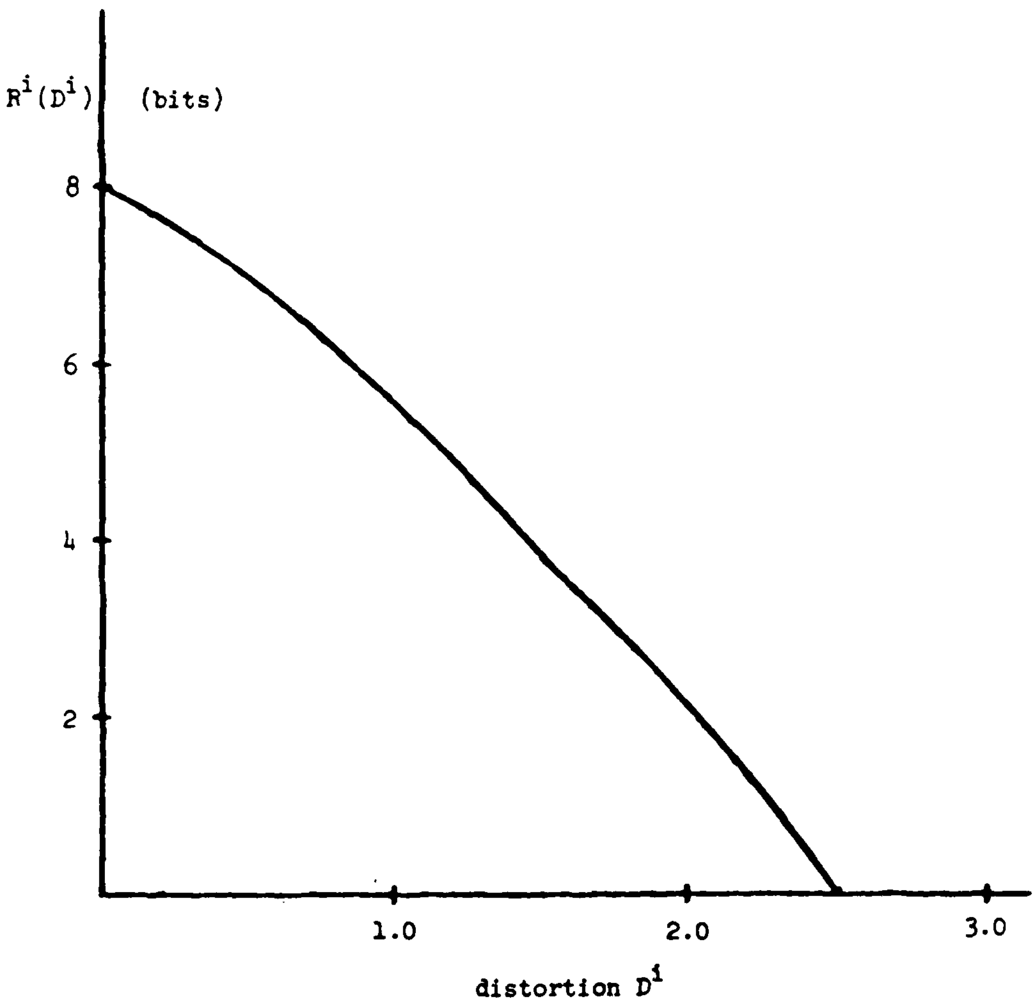

Figure 4.3 The unnormalized link rate distortion function $R^i(D^i)$ for the binary level distortion measure with message priority $P = 2$

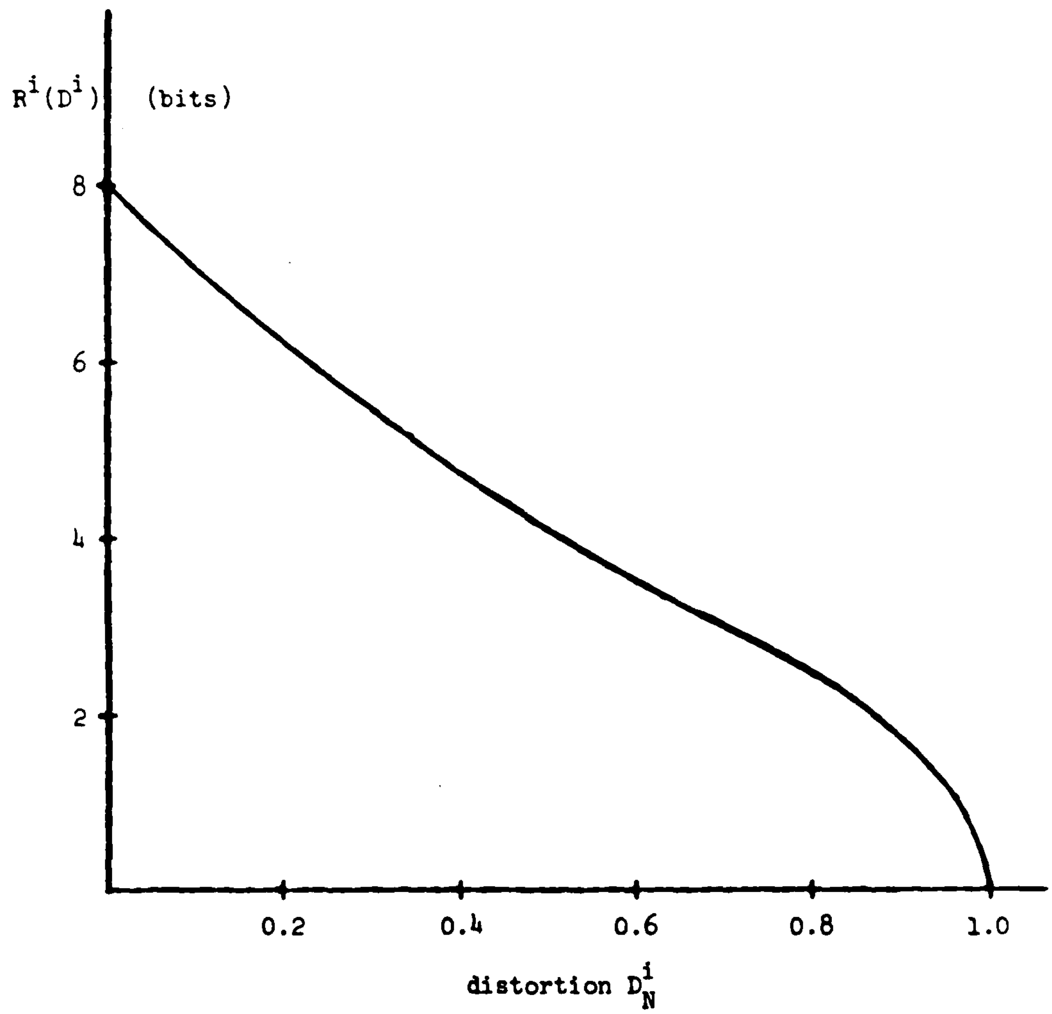

Figure 4.4 The normalized link rate distortion function $R^i(D^i)$ for the binary level distortion measure with message priority $P = 2$

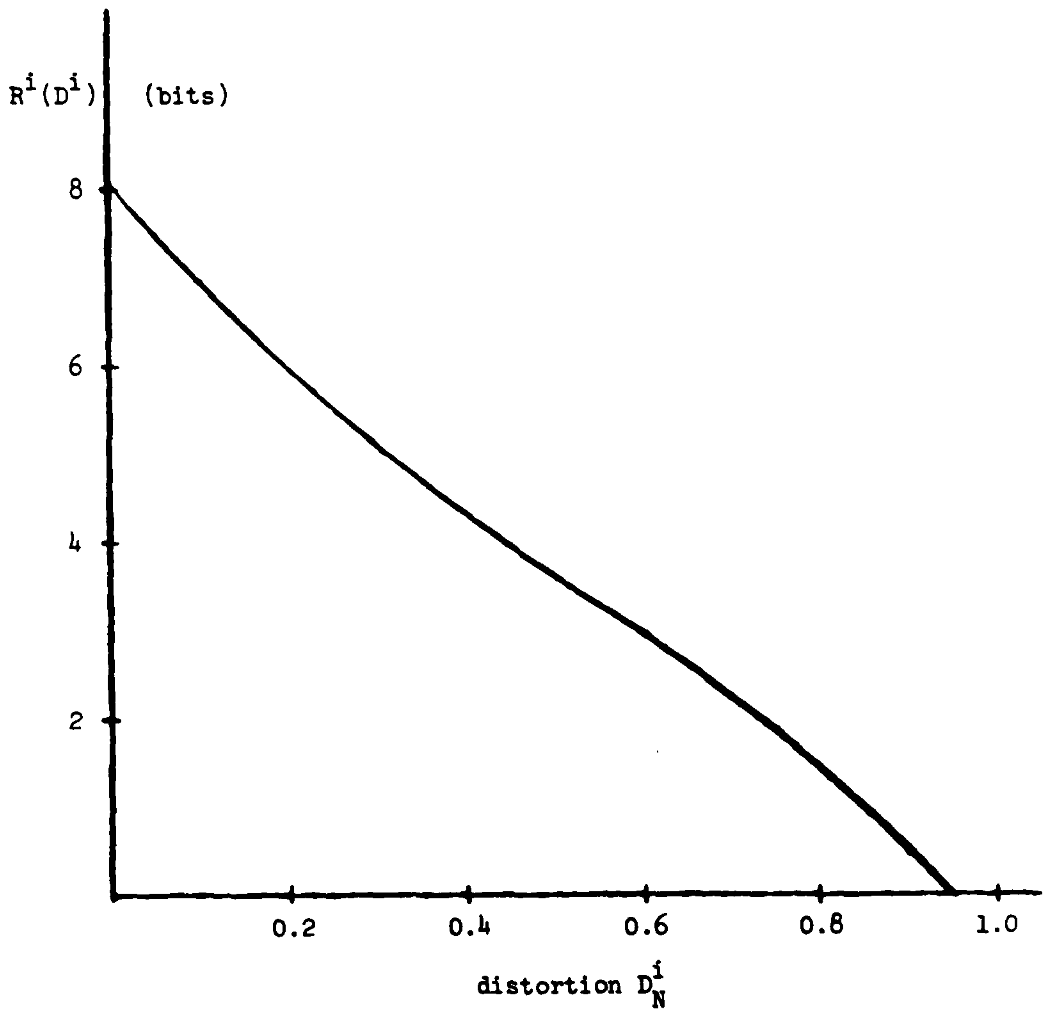

Figure 4.5 The unnormalized link rate distortion function $R^i(D^i)$ for the character level distortion measure with message priority P = 0

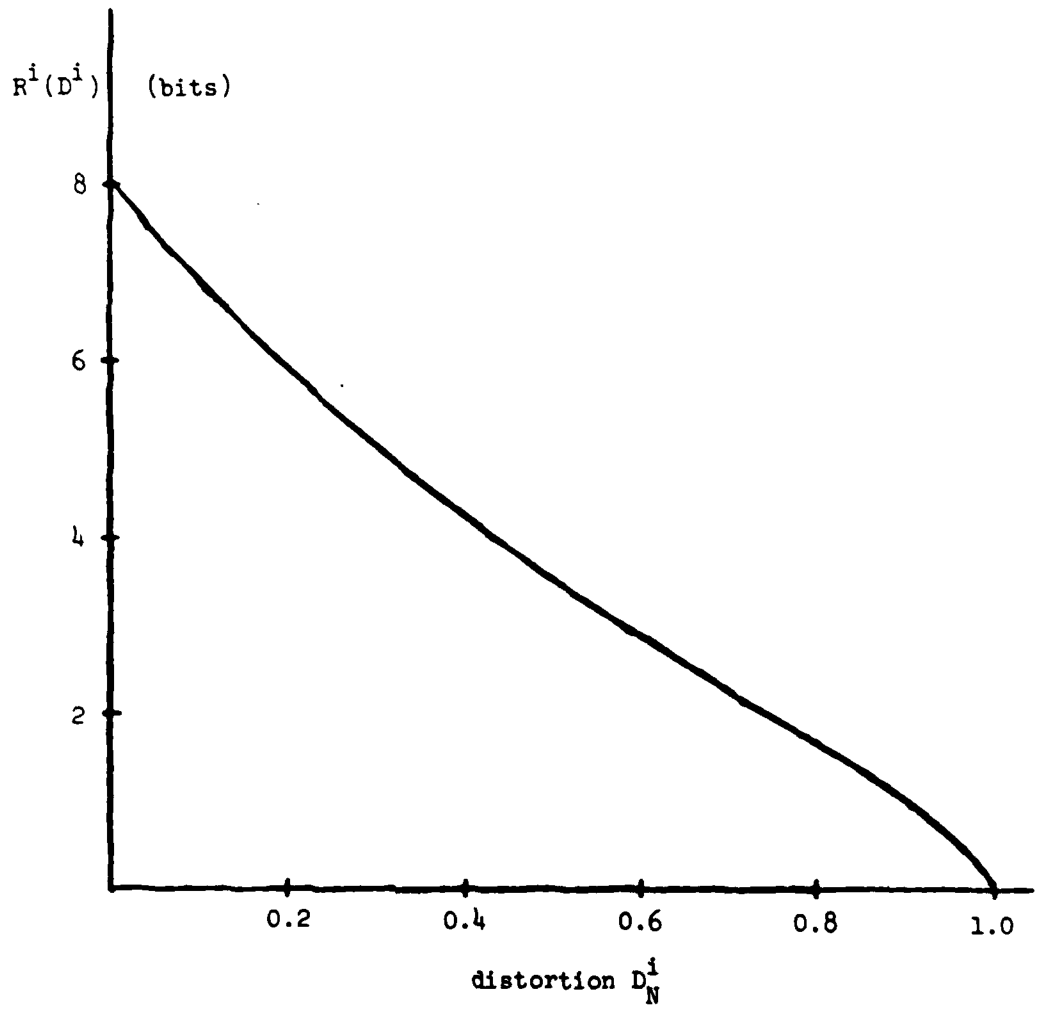

Figure 4.6 The normalized link rate distortion function $R^i(D^i)$ for the character level distortion measure with message priority $P = 0$

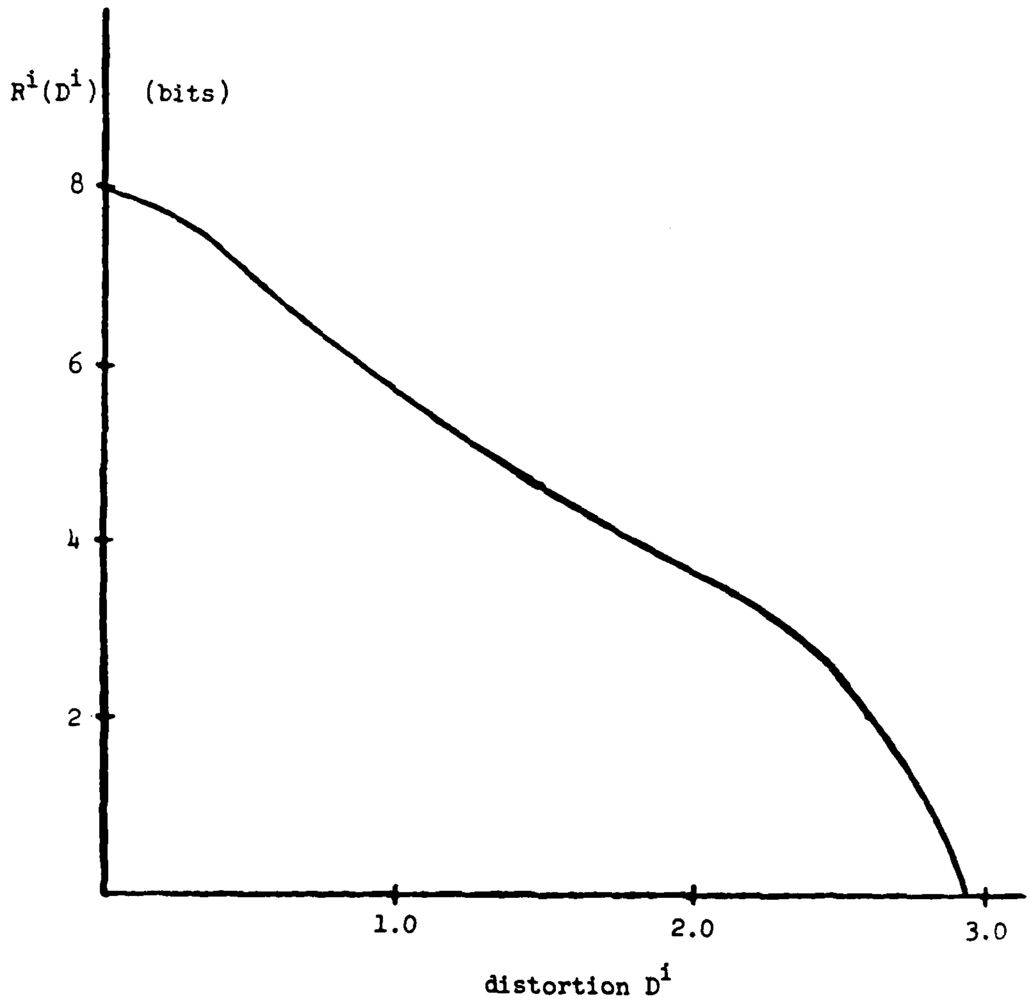

Figure 4.7 The unnormalized link rate distortion function $R^i(D^i)$ for the character level distortion measure with message priority $P = 2$

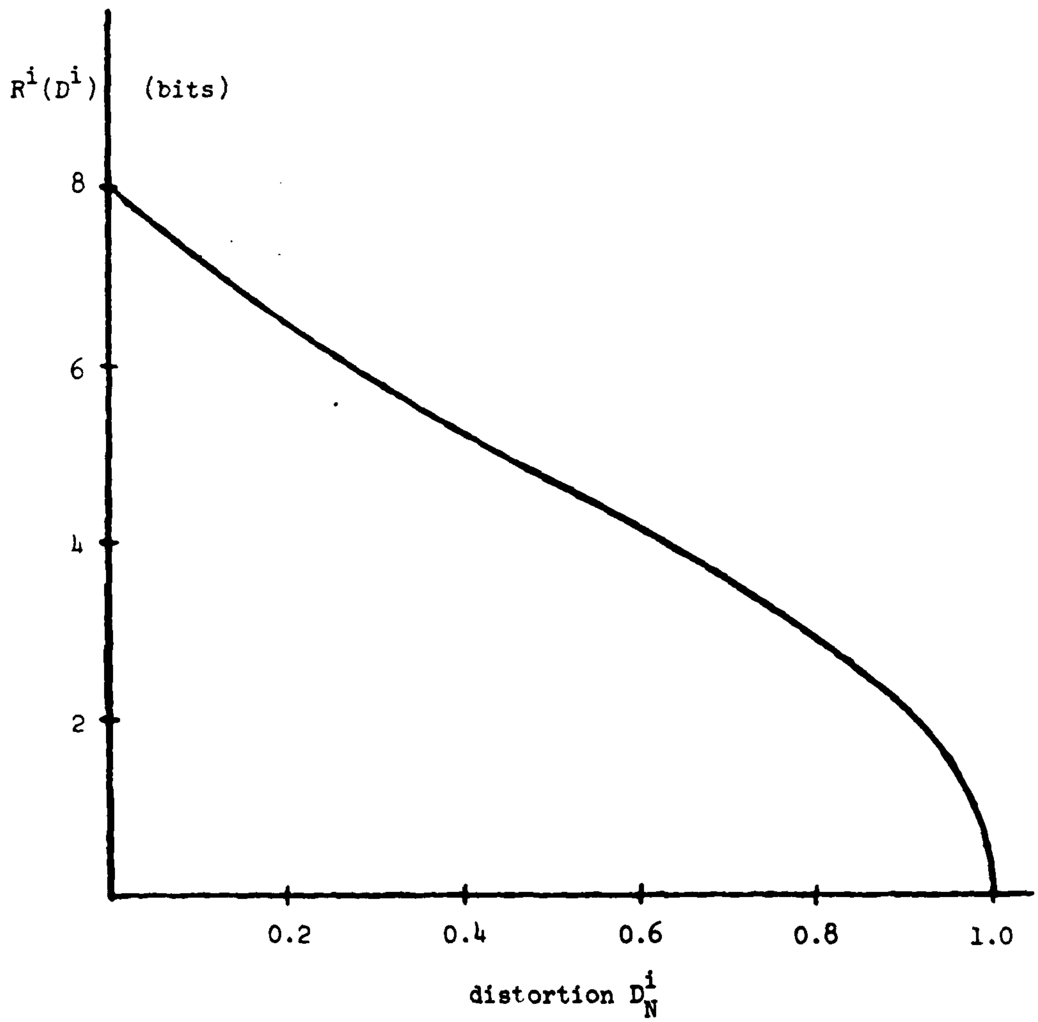

Figure 4.8 The normalized link rate distortion function $R^i(D^i)$ for the character level distortion measure with message priority $P = 2$

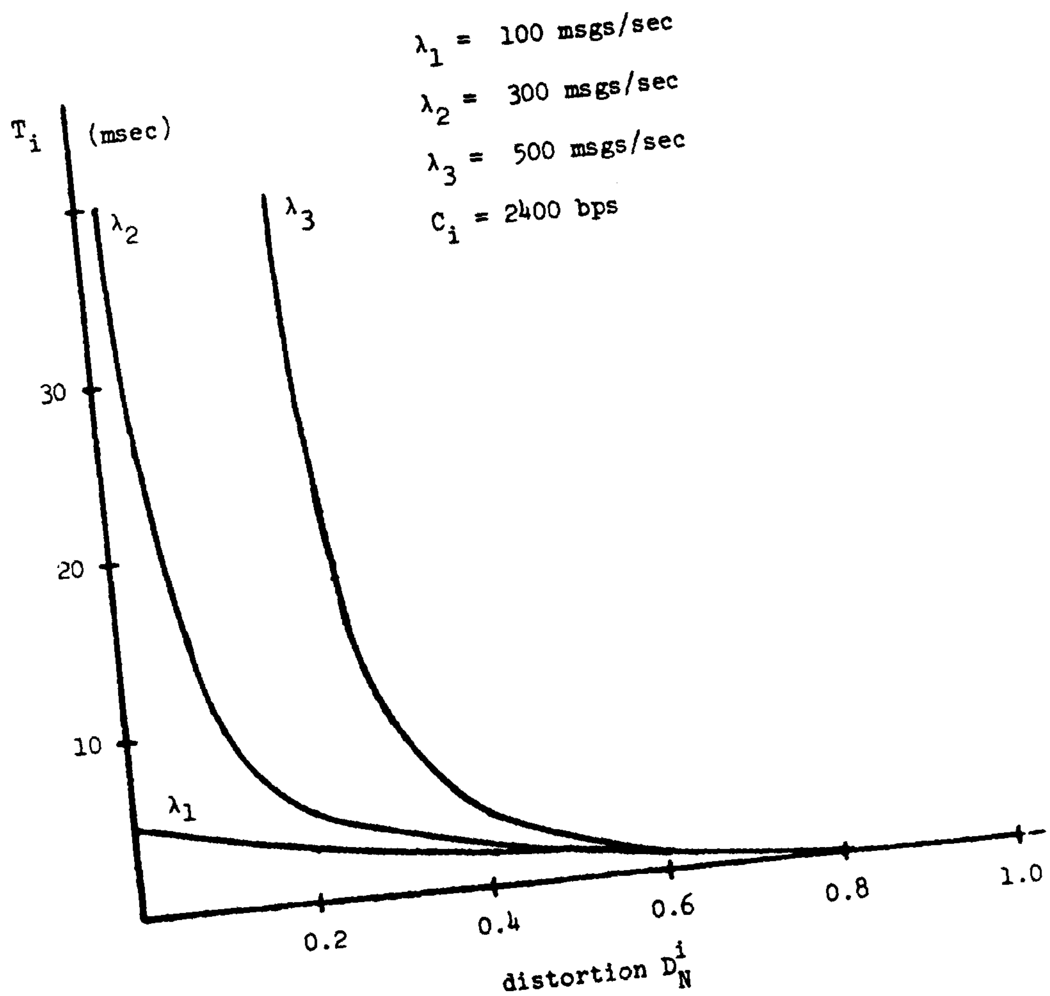

Figure 4.9 Link delay T_i as a function of link distortion D_N^i for the binary level distortion measure with message priority P = 0

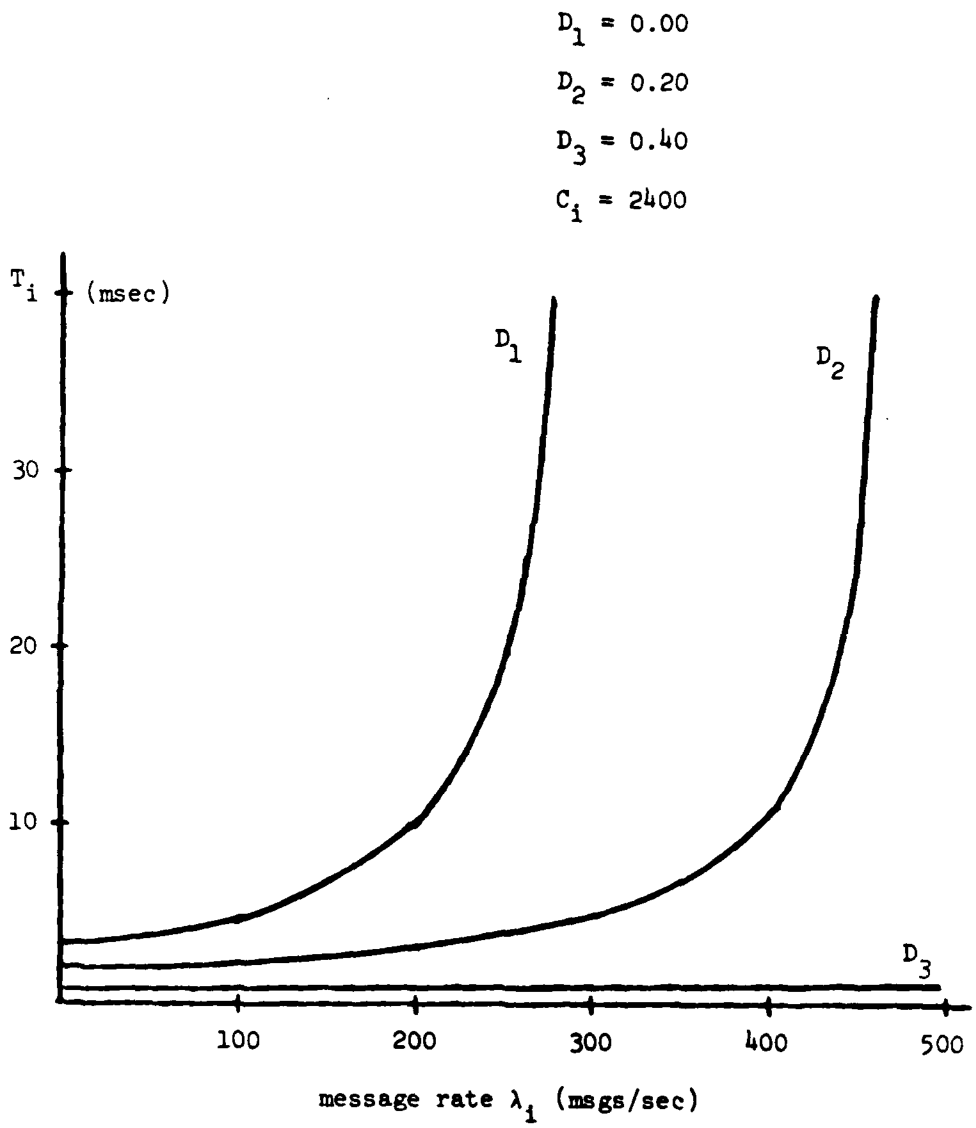

Figure 4.10 Link delay T_i as a function of link message rate λ_i for the normalized binary level distortion measure with message priority P = 0

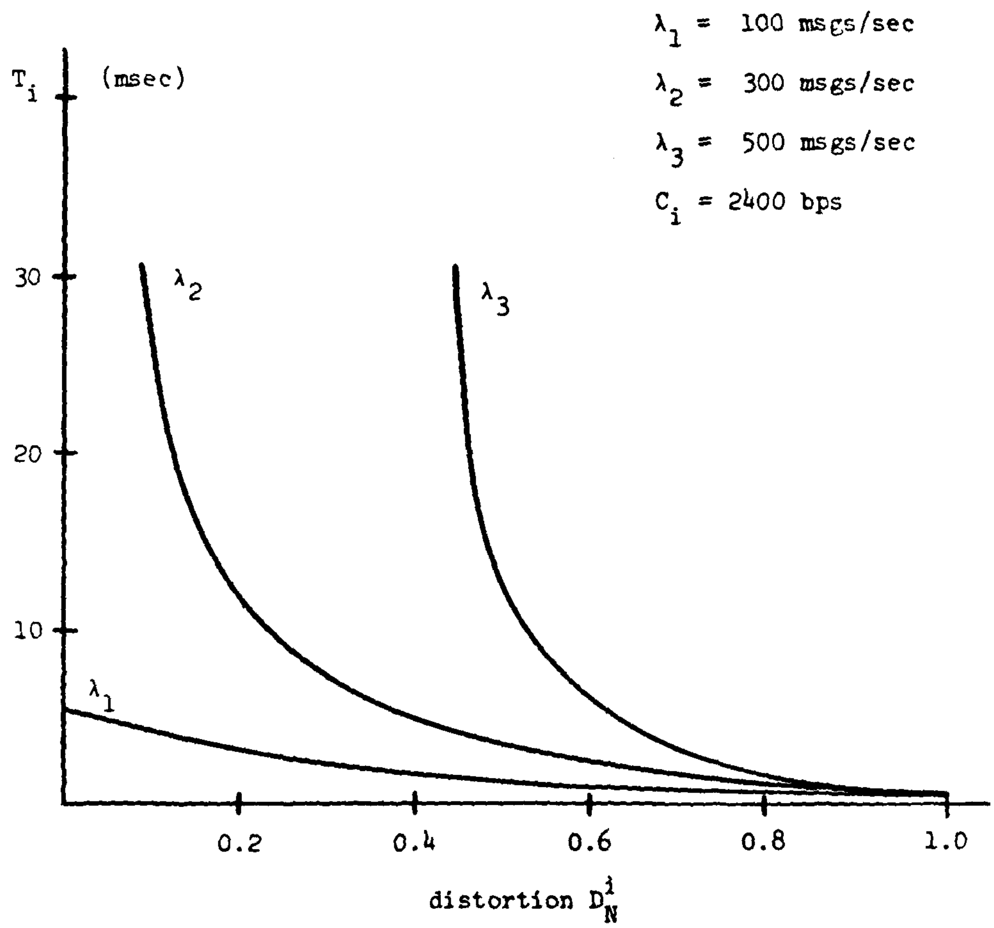

Figure 4.11 Link delay T_i as a function of link distortion D_N^i for the binary level distortion measure with message priority P = 2

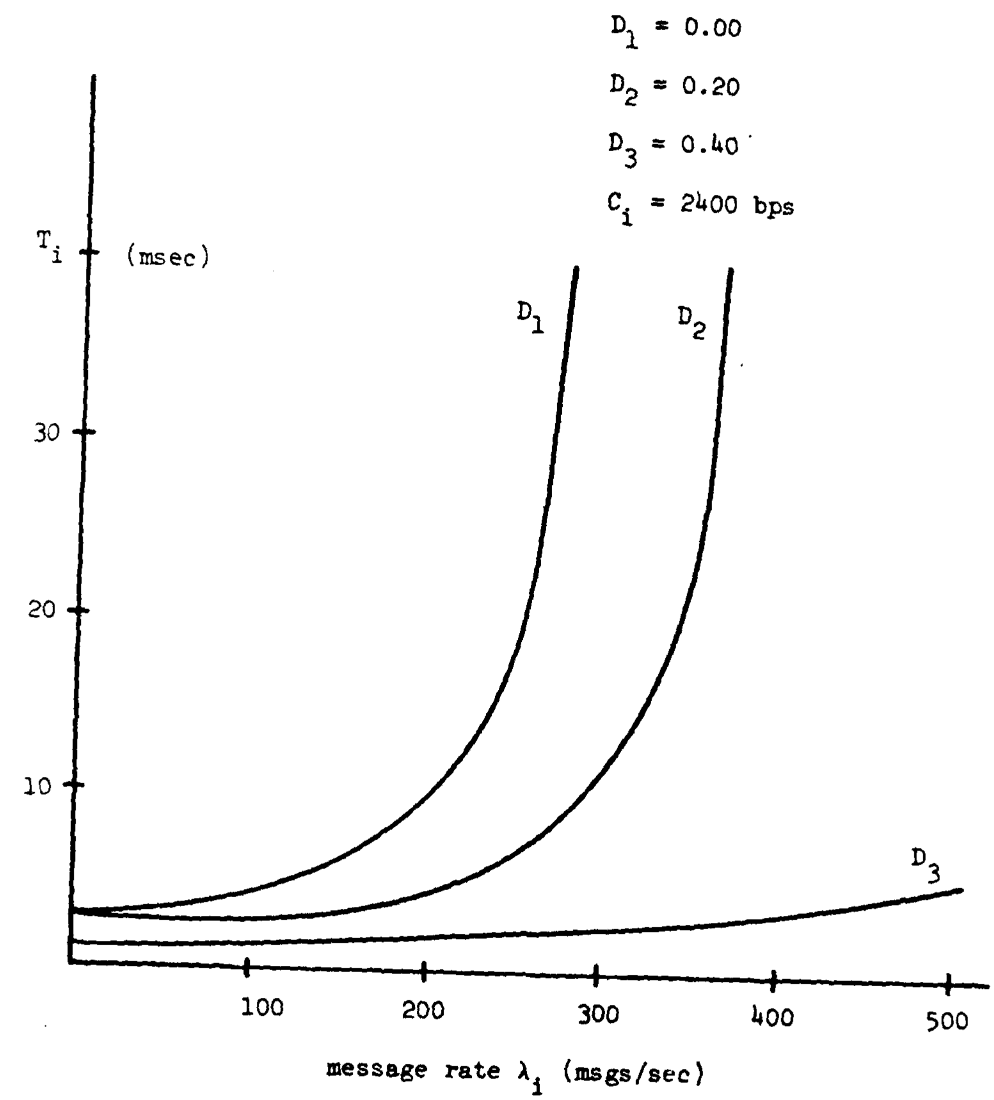

Figure 4.12 Link delay T_i as a function of link message rate λ_i for the normalized binary level distortion measure with message priority P = 2

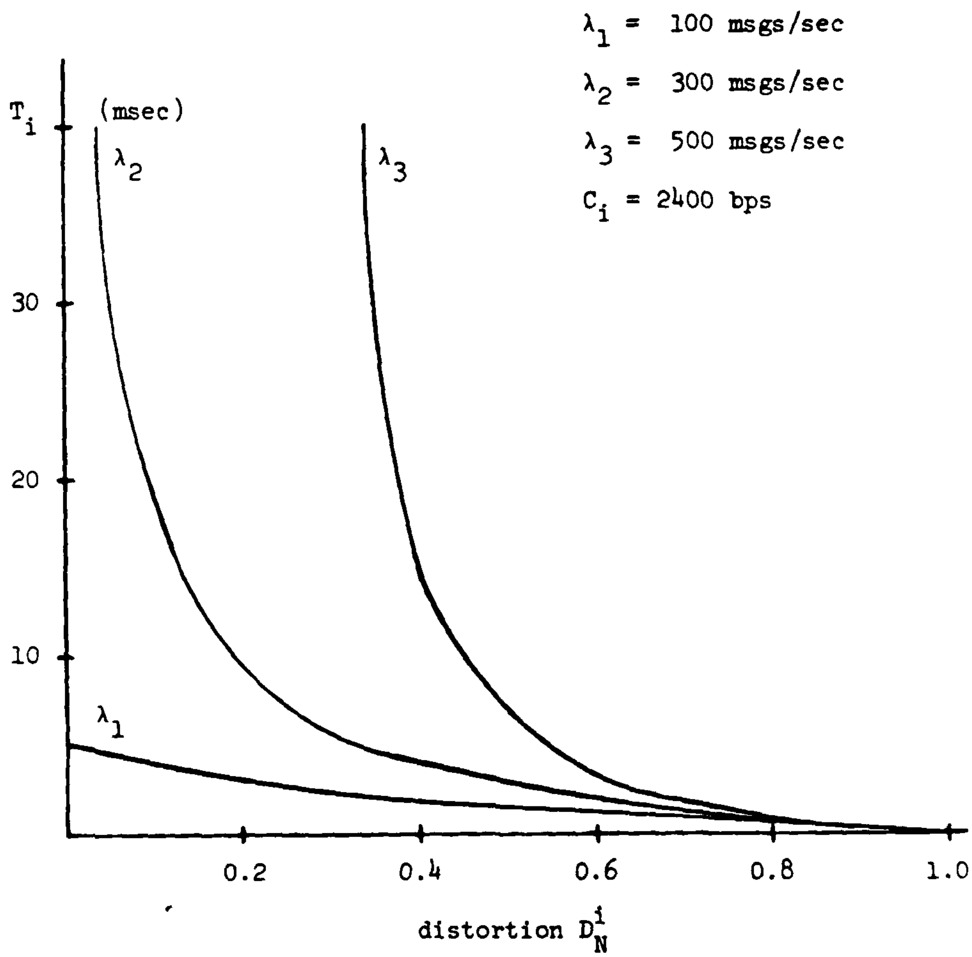

Figure 4.13 Link delay T_i as a function of link distortion D_N^i for the character level distortion measure with priority P = 0

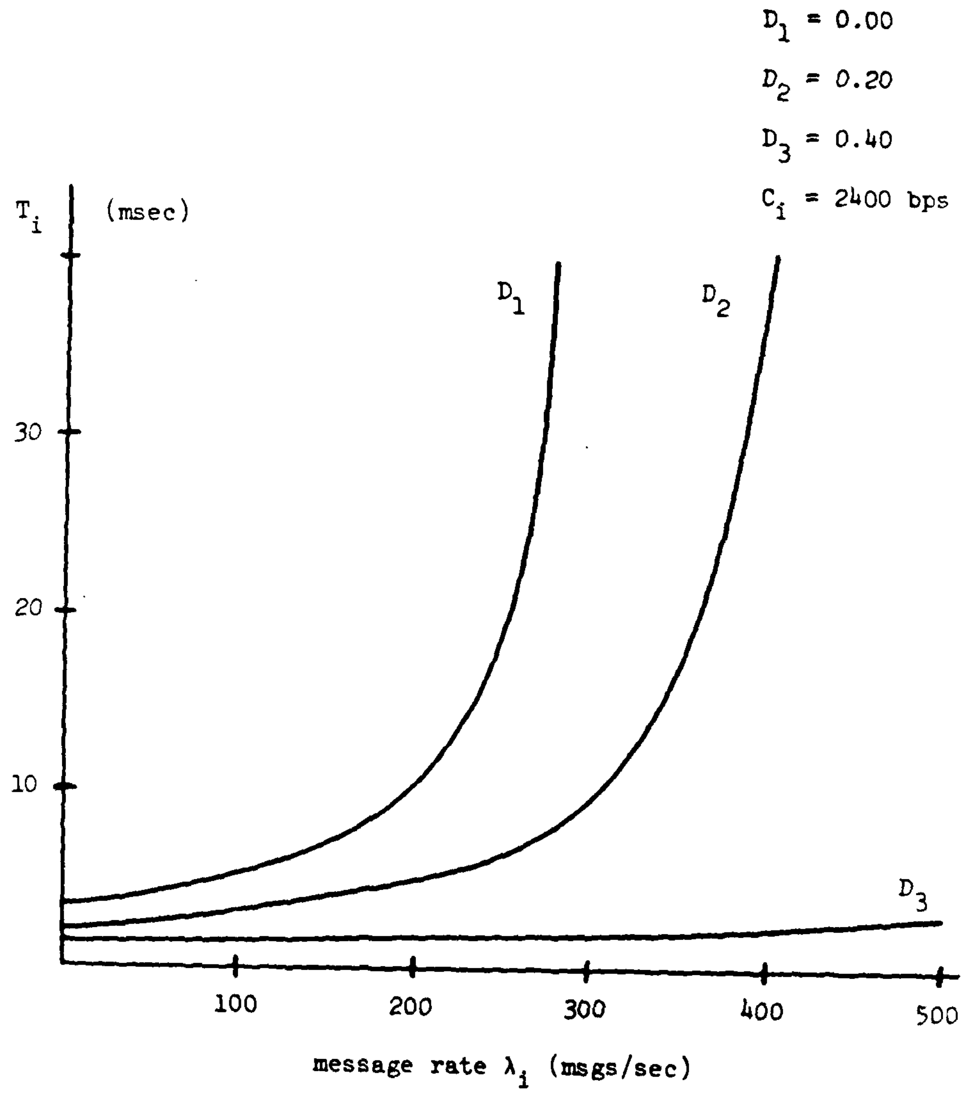

Figure 4.14 Link delay T_i as a function of link message rate λ_i for the normalized character level distortion measure with message priority P = 0

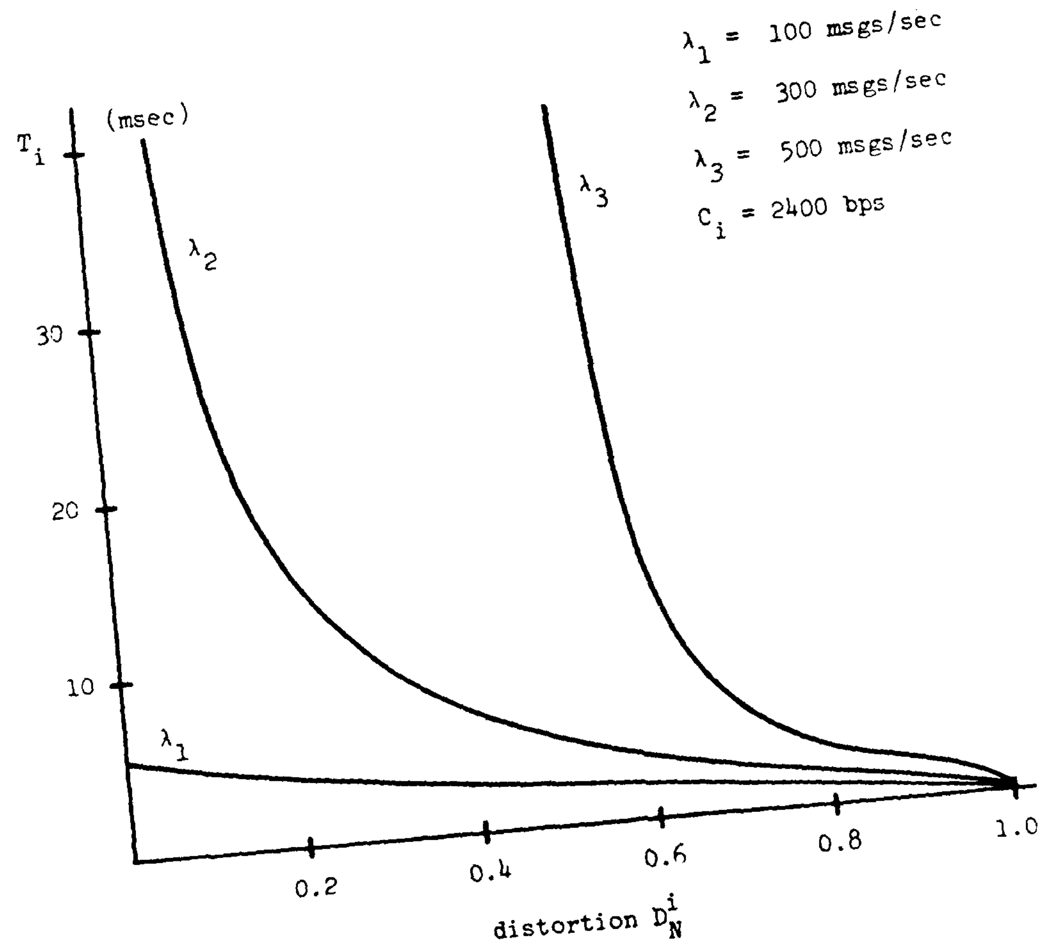

Figure 4.15 Link delay T_i as a function of link distortion D_N^i for the character level distortion measure with message priority P = 2

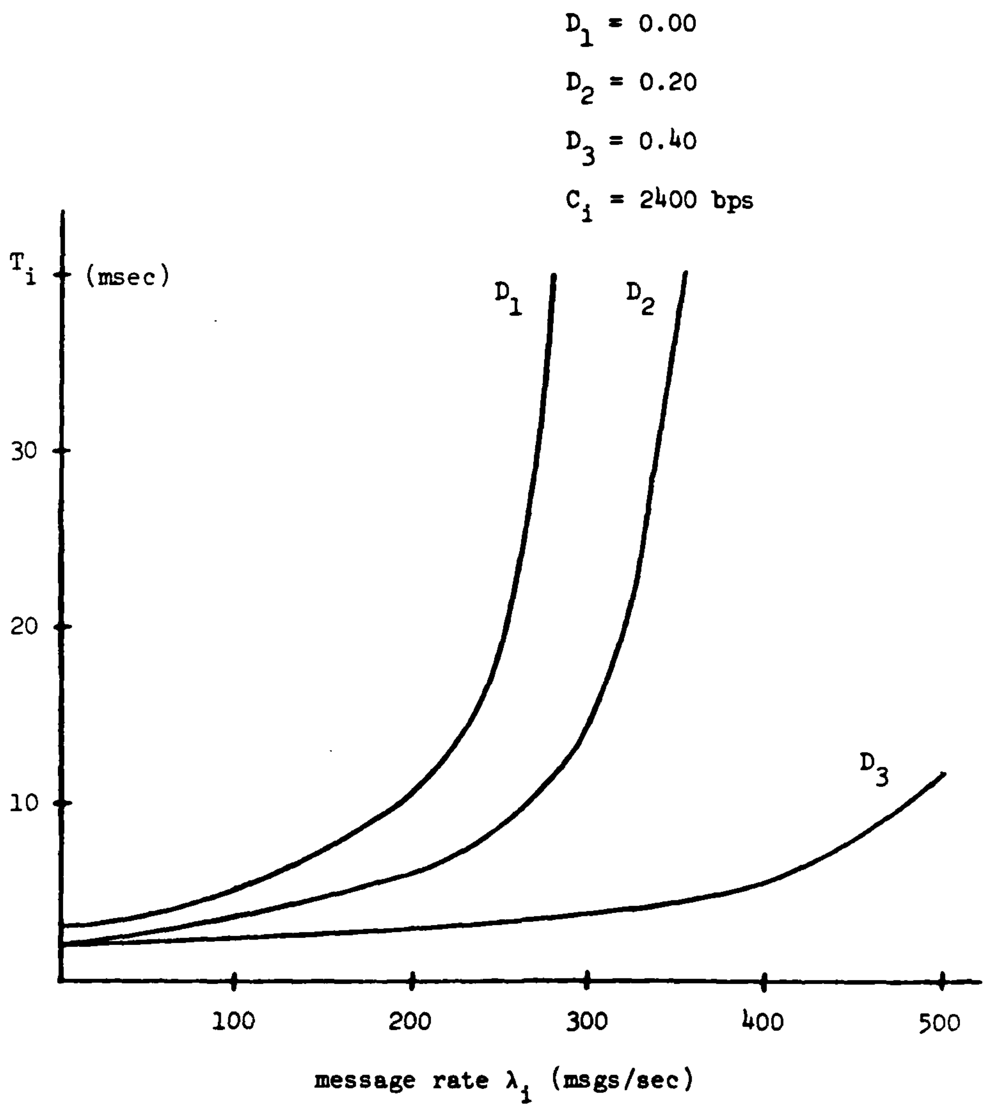

Figure 4.16 Link delay T_i as a function of link message rate λ_i for the normalized character level distortion measure with message priority P = 2

In the next section, the network performance vis-á-vis message delay will be examined as a function of message fidelity.

4.3 Link Delay as a Function of Distortion and Message Rate

In this section, the link delay T_i is calculated for the distortion measures introduced in Chapter 2. The results are presented in graphical form with link delay as a function of normalized distortion D_N^i or message rate λ_i. For all cases, the normalized link distortion D_N^i was used to facilitate performance comparisons. For the purposes of these numerical calculations, it is assumed that the link capacity is

$$C_i = 2400 \text{ bits/sec (bps)}$$

This refers to the standard capacity which is considered to be the upper limit for distortionless transmission. The equivalent message rate, using the problem formulation and results of the preceding section, is 300 msgs/sec. At a message rate λ_i of 300 msgs/sec the classical queueing-theoretic model would lead to infinite link delays. As noted in Chapter 2, and illustrated in this section, by allowing message distortion, it is possible to transmit at rates greater than 300 msgs/sec and maintain finite link delays.

In Figure 4.9, the link delay has been calculated using Eq. (2.74) and the binary level, message priority P = 0, distortion measure. The delay in milliseconds is shown as a function of link distortion D_N^i and is parameterized for three message rates of 100, 300, and 500 msgs/sec. As noted earlier, by allowing message distortion, transmission above the classical limit is possible, and the link delay will remain finite. Of course, for all cases

$$\lim_{D_N^i \to 1} T_i = 0$$

The calculation is repeated for the same distortion measure and the results for T_i as a function of message rate λ_i are shown in Figure 4.10. T_i, in milliseconds, is parameterized for D_N^i = 0.0, 0.2, and 0.6. Figure 4.11 and Figure 4.12 display T_i as a function of D_N^i and λ_i, respectively, for the binary level distortion measure with message priority P = 2. Comparing the results for the zero priority and priority P = 2 binary-level measures reveals that either under equivalent distortion or equivalent message rate, the link delay is slightly greater for the priority P = 2 measure. This result could be predicted from the calculations of the preceding section and highlights the expected performance tradeoff between message distortion and channel delay.

The calculations using the character level distortion measure, with priority P = 0 and priority P = 2, yield the curves of Figures 4.13 thru 4.16.

Again, the link delay, under equivalent distortion or message rate conditions, is slightly greater for the priority P = 2 measures. In addition, the delay under the character level distortion measures is slightly greater than the delay, under equivalent conditions, of the binary level measures. As noted earlier, these contrasting features highlight the expected tradeoff between message distortion and channel delay.

The results presented in this section may be concisely summarized:

1. Distortion measures with message priority P = 2 result in higher channel delays than measures with priority P = 0.
2. The binary level distortion measure results in lower channel delay than the comparable character level distortion measure.

In the next section, a network example is developed to illustrate (j,k) path level rate distortion and the effect on average end-to-end delay and average network message delay.

4.4 Network Example

In this section, a network example is presented to illustrate the implementation of the path level relationships introduced in Chapter 2. The effect of the path level relationships on the Min-Hop routing doctrine, the use of the RDF Decomposition Theorem, and a new upper bound on end-to-end delay are examined with respect to the network example. In particular, end-to-end delay constraints are considered as functions of distortion and are upper-bounded, and the average network message delay T is calculated. The results are displayed as a locus of possible delay-distortion pairs.

The network used in Section 3.2.3 to illustrate the Min-Hop algorithm is the basis for the example of this section. The network is reproduced in Figure 4.17, where the underlined values are the link capacities μC_i in messages/sec. The assumed traffic requirements as before, are $\gamma_{ab} = \gamma_{cd} = 10$, $\gamma_{bh} = \gamma_{eb} = \gamma_{fb} = \gamma_{gf} = 5$, and $\gamma_{ca} = 1$, each in messages/sec. Similarly, the end-to-end average delay constraint values are $T_{ab} = T_{cd} = 1.0$ sec, and α_i are taken to be 0.8 for each link, except link fc which was set at 0.9

Part A of the Min-Hop algorithm led to the flow assignment given in the figure by the first number in parentheses for each link. The corresponding routing was

γ_{ab}: 8 via aedb 2 via afdhb	γ_{eb}: 5 via edhb
	γ_{fb}: 5 via fcgb
γ_{cd}: 10 via cfd	γ_{gf}: 5 via ghf
γ_{bh}: 5 via bdh	γ_{ca}: 1 via cfa

in messages/sec, the average path length was $\bar{n} = J = 2.537$, and the two end-to-end average delays were $T'_{ab} = 1.14$ and $T'_{cd} = 1.33$ sec.

Following the problem formulation of the previous section, it is assumed that each source in the network can choose from among 256 equiprobable messages. For the purposes of the example, it is assumed that the network is operating under the binary level distortion measure with message priority P = 0 for all commodities.

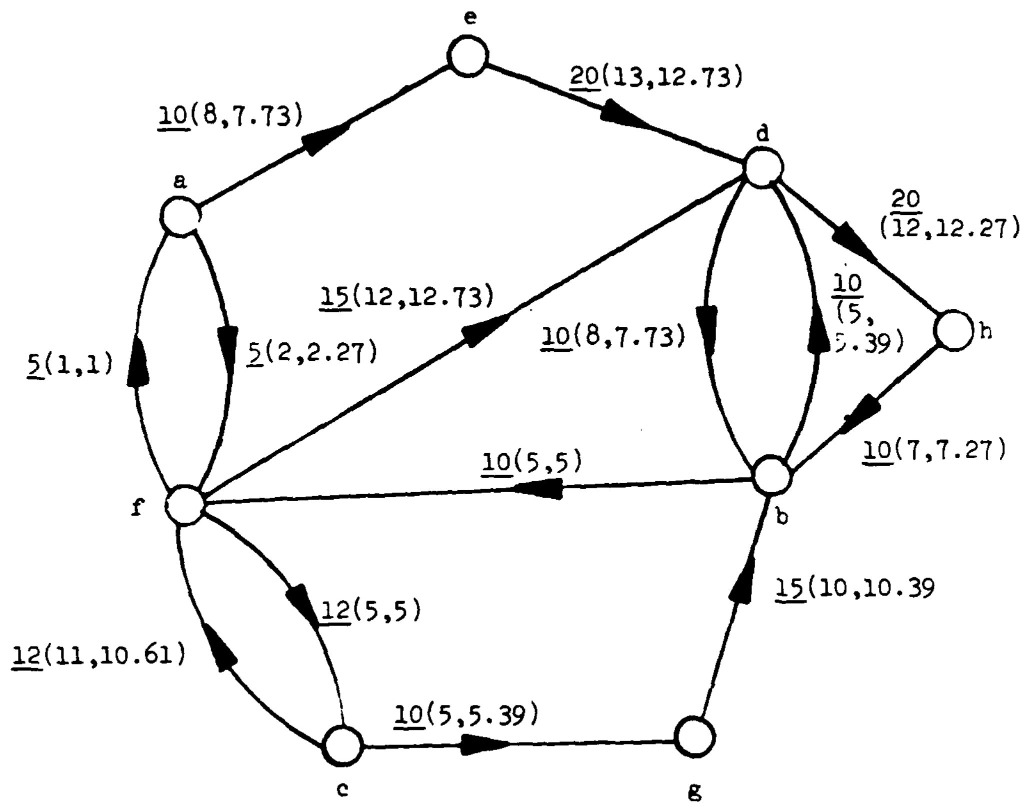

Figure 4.17 Network example for centralized Min-Hop algorithm constrained by average end-to-end message delay

The objective of the first calculation is to express the end-to-end delay for the (a,b) traffic as a function of the path level distortion, i.e.; $T_{ab} = T_{ab}(D_{ab})$ where D_{ab} is distortion for the messages originating at the source node a with node b as their destination. It is assumed further that a network optimization goal is to minimize message corruption. As shown in Chapter 3, the optimization goal can be achieved when routing assignments are made according to the Min-Hop algorithm. Therefore, the initial feasible flow that obtains from the Min-Hop algorithm is assumed for this example. Because of the equiprobable message distribution at source node and the symmetry of the distortion measure, the output messages are also equiprobable (as shown in [1,3]). Hence, the link level distortion measure for the (a,b) traffic is equal to the path level distortion measure if the (j,k) traffic on path π_{ab} is transmitted with the same distortion as the (a,b) traffic. Of course, this sets a limit to the minimum distortion possible for the (j,k) traffic.

The result of the calculation is shown in Figure 4.18. $T_{ab}(D_{ab})$ is equal to 1.14 seconds when the messages are transmitted with zero distortion. As the message distortion increases T_{ab} decreases. By allowing a cumulative message distortion $D_{ab} = 0.02$ the end-to-end delay can be reduced to 1.0 sec which is the assumed constraint requirement for this example. Also shown in Figure 4.18 is an upper bound for the end-to-end delay T_{ab} calculated as a function of distortion. The upper bound

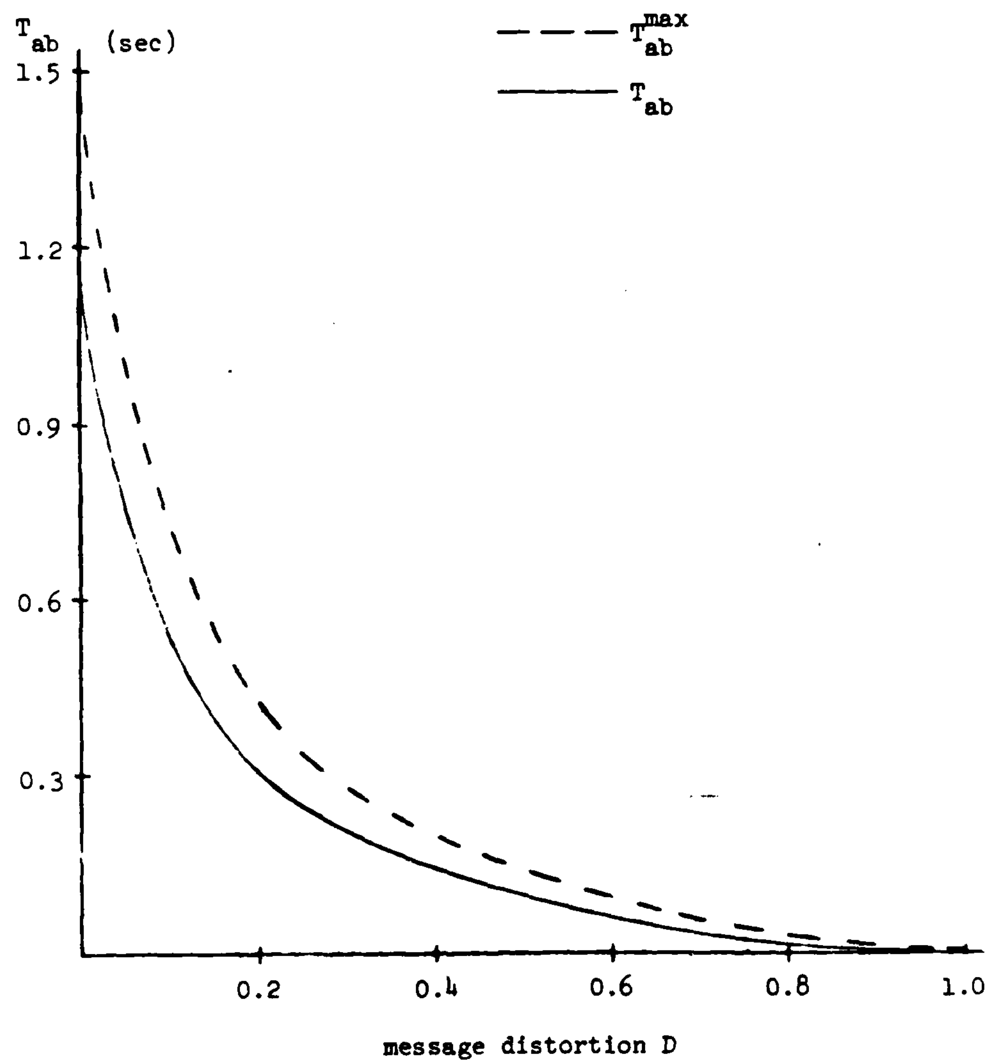

Figure 4.18 The end-to-end delay T_{ab} and the upper bound T_{ab}^{max} as functions of message distortion

was found by bounding each of the alternate routings. Letting T_i^ℓ be the ith link delay on the ℓth alternate routing and denoting the number of links on the ℓth route by n_ℓ, the upper bound for the end-to-end delay is given by

$$T_{sd}^{max} = \sum_{\substack{\ell \\ i\epsilon\pi_{sd}^\ell}} r_\ell(s,d) n_\ell \max_i \{T_i^\ell\} \qquad (4.7)$$

where s is the source node and d is the destination node, and the summation is over all the ℓth alternate routings and the ith links on each path π_{sd}^ℓ. It is clearly the case that $T_{ab}^{max} \geq T_{ab}$ for all distortions D_{ab} in the range (0,1); it is also clear that the calculation of T_{ab}^{max} is always significantly easier than the calculation of T_{ab}.

Finally, this section concludes with the calculation and presentation of the average network message delay T(D) as a function of the link level distortion

$$T(D) = \frac{1}{\gamma} \sum_{i=1}^{M} \frac{\lambda_i R^i(D^i)}{C_i - \lambda_i R^i(D^i)} \qquad (4.8)$$

where

$$D^i = D$$

for all links and

$$\gamma = \sum_{j,k} \gamma_{jk}$$

This is the sort of message delay that can be obtained if all links are allowed distortion D. The consequences for the end-to-end message distortion would depend on the role of the switching computers. If the switching computers did not participate in error detect/correct processing, the message distortion would, in general, accumulate. The exception of course, is in the case of equiprobable input message distributions and a symmetrical distortion measure, as in this example. If the switching computers do participate in error detect/correct processing, the message distortion would not accumulate as the messages move through the network, but there could be significant delays due to the additional processing. The average message delay for this network example is presented in Figure 4.19. The average delay for messages with zero distortion is 0.861 seconds. As expected, in the limit of total message corruption the average delay goes to zero.

In the next and last section, the chapter concludes with a comparison of the best and worst case results for message delay as a function of message distortion.

4.5 Conclusion

In an earlier section, message delay was calculated for a number of distortion measures. In Figure 4.20, the best case (least delay) and worst case (most delay) results are compared. The best case result was obtained under the binary level distortion measure with message priority P = 0. The worst case result was obtained under the character level distortion measure with message priority

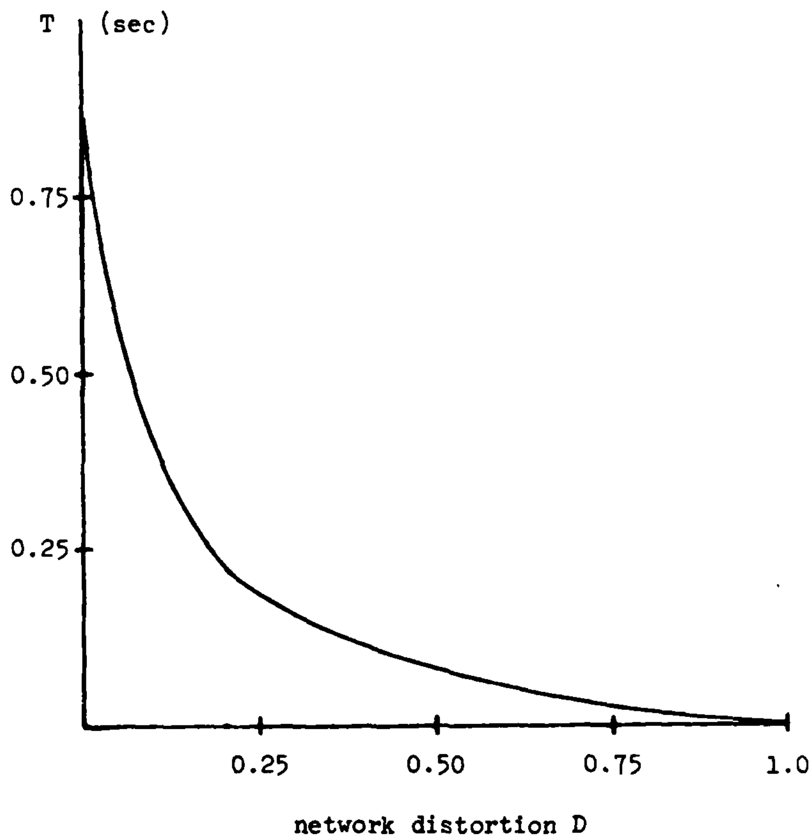

Figure 4.19 Average network message delay T as a function of network message distortion D

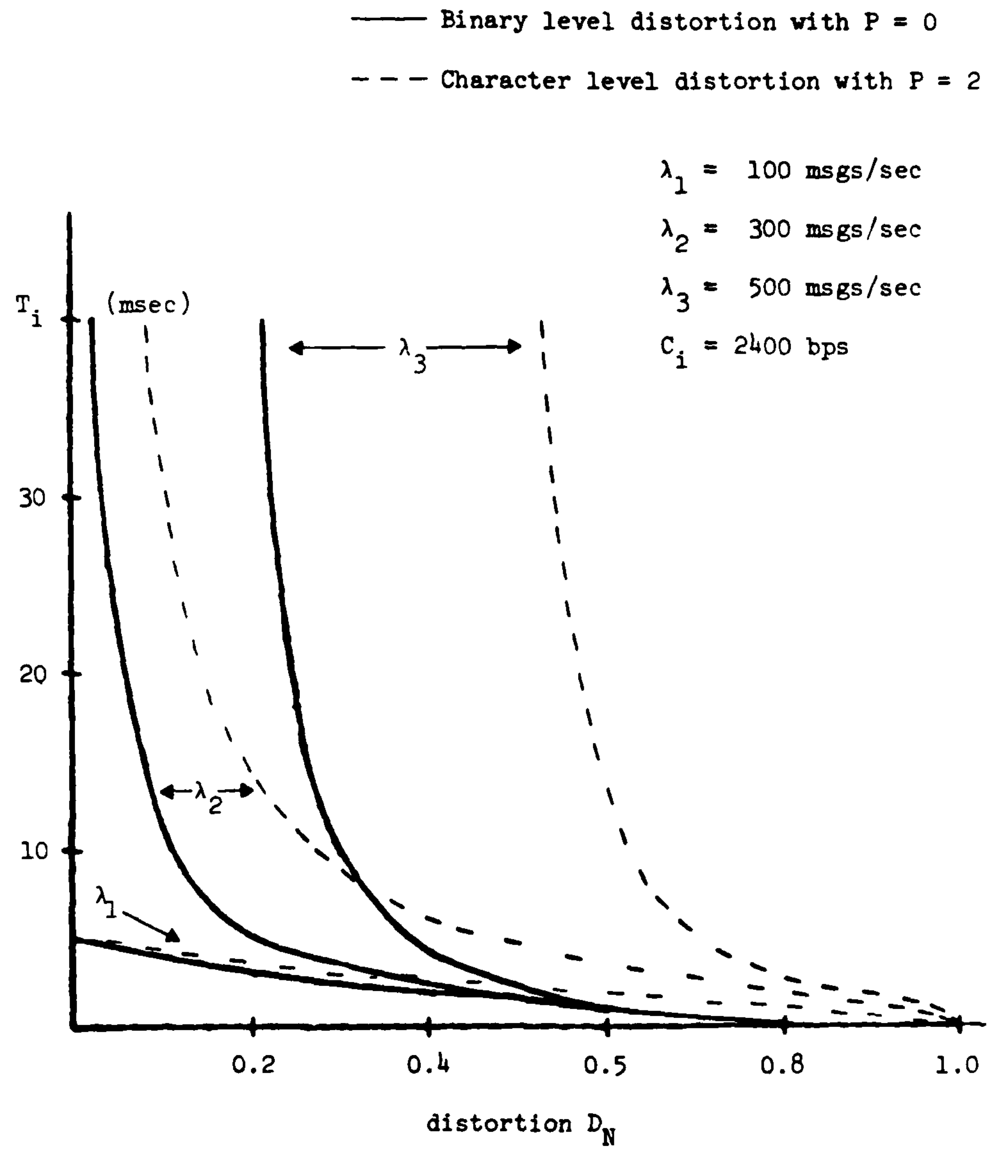

Figure 4.20 Link delay T_i compared for normalized binary and character level distortions with P = 0 and P = 2 respectively

P = 2. As can be seen from Figure 4.20, the difference in delay for the two results varies greatly as the distortion ranges from 0 to 1. Again, the implication is that message delay can be reduced by allowing message corruption.

In this chapter, results from Chapters 2 and 3 were applied to a network example. The example provided a demonstration of the thesis that message delay can be reduced by allowing message distortion, thereby increasing the effective network channel capacity.

Chapter 4 Notes

1. A. J. Viterbi and J. K. Omura, Principles of Digital Communication and Coding, New York: McGraw-Hill, 1979.

2. C. E. Shannon, "Coding theorems for a discrete source with a fidelity criterion." IRE Nat. Conv. Rec., Part 4, pp. 142-163, 1959.

3. T. Berger, Rate Distortion Theory, Englewood Cliffs, New Jersey: Prentice-Hall, 1971.

4. R. G. Gallager, Information Theory and Reliable Communication, New York: Wiley, 1968.

5. J. W. Cohen, The Single Server Queue, New York: Wiley, 1969.

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Chapter 5

CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

5.1 Conclusions

The objective of this research has been to provide a theoretical model for dealing with the problem of message corruption in large-scale, data-communication networks. The message corruption can be due to link noise, finite nodal buffers, and protocol dependent errors. Information-theoretic and queueing-theoretic methodologies have been used in the articulation and analysis of the theoretical model with several network examples serving to illustrate specific implementation schemes and provide computational results.

In Chapter 1, a brief overview was presented along with the stochastic 'network of queues' model. Chapter 2 contained the basic information-theoretic concepts relating to rate distortion theory and their application to data-communication networks. Chapter 3 introduced and analyzed several new routing doctrines whose assignment policies optimized message fidelity. Chapter 3 also provided several network examples to illustrate the Min-Hop routing algorithm in both its centralized and decentralized forms, with and without message delay constraints. In Chapter 4, the rate distortion theory of Chapter 2 and the Min-Hop algorithm of Chapter 3 provided the basis fo· computational results demonstrating

The effects of the various distortion measures and their application to network traffic.

The fundamental conclusions of this research are:

1. Rate distortion theory can be extended and enhanced to provide a technique for network performance analysis in the event that message errors are an important consideration.

2. Message delay may be formulated as a function of message corruption, in addition to the traditional variables of message length, link flow, and link capacity.

3. The Min-Hop routing policy optimized network performance in the noisy message environment and can be implemented in a centralized or distributed fashion, with and without message delay constraints.

There are, of course, philosophical questions regarding the utility of a system implementation. The cost of successive source encodings as a message is passed over a series of links may offset the decrease in message delay and is certainly a tradeoff to be considered.

5.2 Future Research

There are a number of interesting questions raised by these investigations which could serve as a motivation for future research. In the domain of the network level rate distortion theory, the effect and construction of additional distortion measures should be considered, e.g. distortion metrics which are functions of message context. In a similar fashion, a small scale network in which message rate and message distortion can be controlled would provide an invaluable tool for validation of theoretical results. An associated problem is the relation of these investigations, especially the network message delay results, to bit oriented and character oriented protocols. And, of course, there is need to develop an optimum distributed Min-Hop algorithm which is also viable from an implementation perspective. Finally, as an improvement to the 'network of queues' model would be the use of the Pollaczek-Khinchin formula for average waiting time in an M/G/1 system and the network level rate distortion theory to develop a M/G/1-RDF network theory.

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